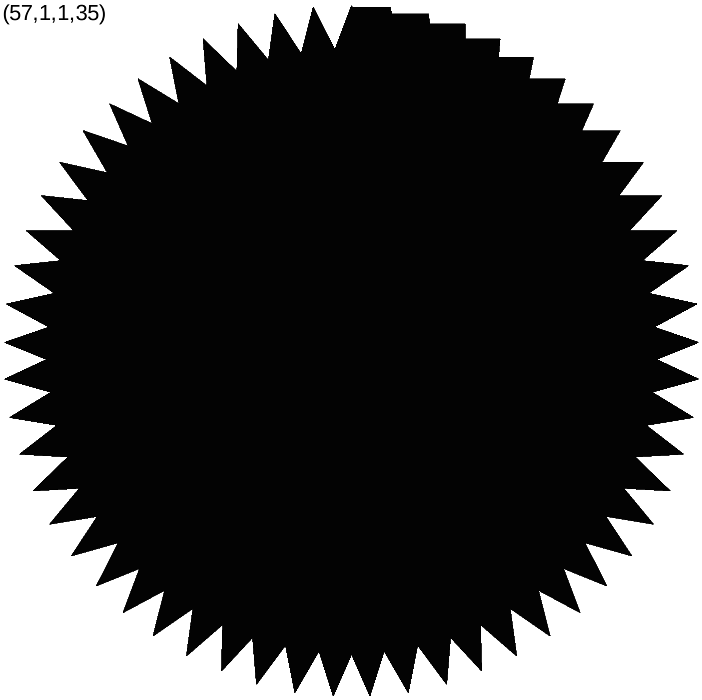
Examples of Modular Multiplicative Inverses

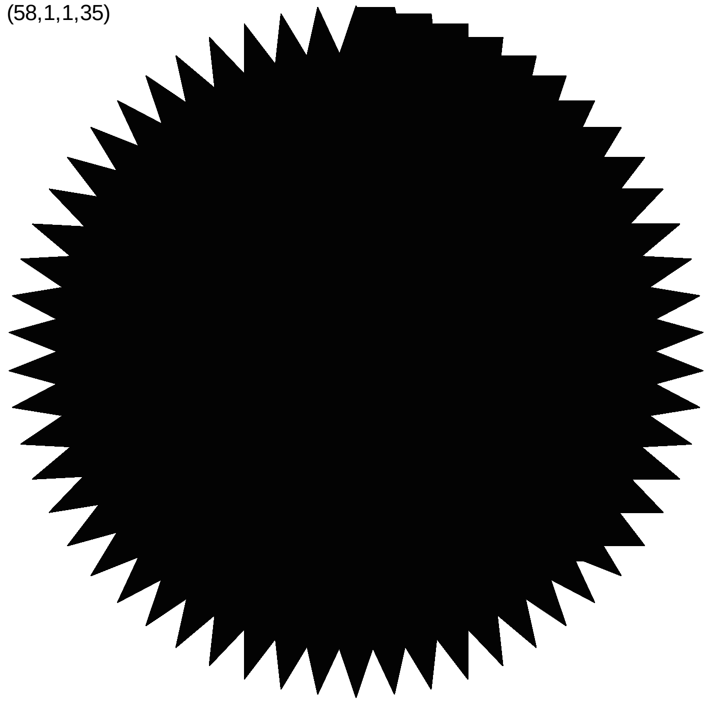
The star on the left has 57 points and the one on the right has 58 points but all in all they look very similar to one another. Beneath each image, the first 5 segments of how each star is drawn is noted in **red**. Each appears to be a pentagram cracked open a bit rather than closed so that subsequent sets of 5 continue making open pentagrams.

If you watch the image on the left get created (using *Toggle Drawing* once you click on the link), the image is one of a pentagram bouncing around a lot, but the image on the right shows a much simpler pattern, the pentagram rotates around one vertex at a time until the image is completed. The fifth segment ends after 175 = 5·35 vertices have been counted in both images. To get to this endpoint all vertices have been counted three times in both cases (171 = 3·57 and 174 = 3·58) and the 5th endpoint is at vertex 4 = MOD(5·35, 57) on the left but at vertex 1 = MOD(5·35, 58) on the right. Put another way, ***35 and 5 are modular multiplicative inverses modulo 58*** (or in shorthand, **35 and 5 are MMI MOD 58**).

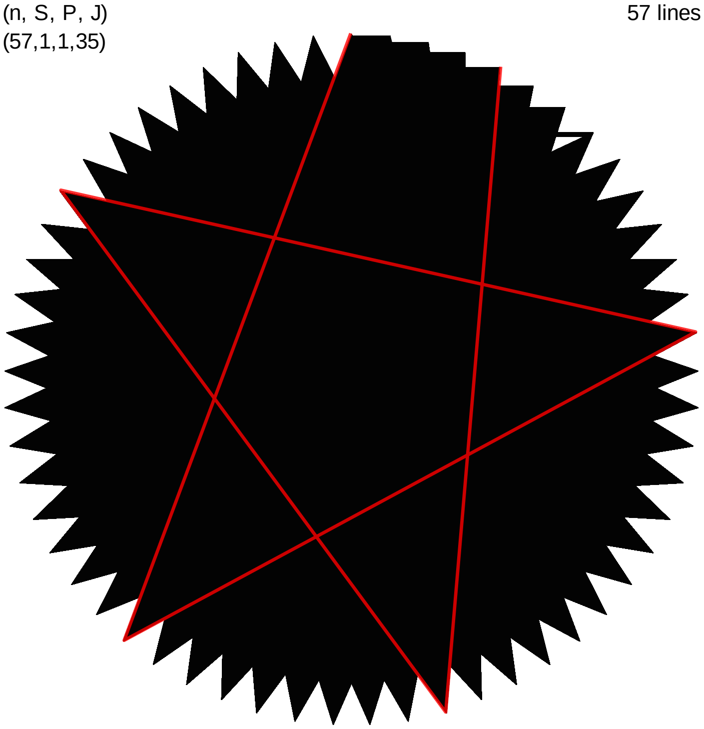
(n, S, P, J) 57 lines
(57,1,1,35)



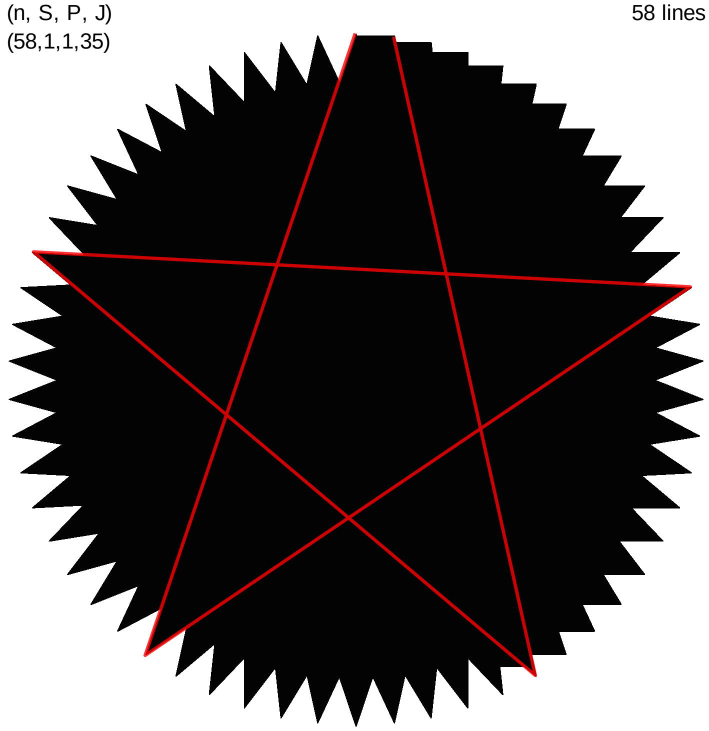
(n, S, P, J) 58 lines
(58,1,1,35)



(n, S, P, J) 57 lines
(57,1,1,35)



(n, S, P, J) 58 lines
(58,1,1,35)



Definition. If two numbers ***a*** and ***b*** > 0 are relatively prime, then there will be a number ***c*** such that ***a·c*** = 1 MOD ***b***. The numbers ***a*** and **c** are said to be **modular multiplicative inverses MOD *b*** (***a*** and ***c*** are MMI MOD ***b***).

Given this, we might expect other 5-sided rotating figures to emerge when ***b*** is coprime to 5. Check that ***n* = 57, *J* = 23** is also a rotating pentagram while ***n* = 56, *J* = 45** and ***n* = 54, *J* = 11** produce stars as rotating pentagons because, in each case, ***J*** is the MMI of 5 MOD ***n***. Similarly, the ***n* = 55, *J* = 37** star is a rotating triangle because 37 is the MMI of 3 MOD 55.

A String Art Example. The two puffy pentagrams shown below are created by "almost-triangles" being rotated around the vertices of the vertex frame. The only difference between the two is that ***P*** = 32 on the left and 33 on the right. Each image has 95 subdivision endpoints (***n·S*** = 95) and 95 segments, and **the first *cycle* is shown in red** below. The left image is created in a special way: note in **red** that each point on the vertex frame is ticked off, **one next to another**. The right image ticks off every 4^{th} vertex point on the vertex frame and *Toggle Drawing* makes it look much more "haphazard." This ticking off of subdivisions on the vertex frame is a consequence of modular arithmetic. In particular, **3 is the MMI of 32 MOD 95** on the left (1 = MOD(3·32, 95) since 3·32 = 96), but on the right, 4 = MOD(3·33, 95) since 3·33 = 99.