

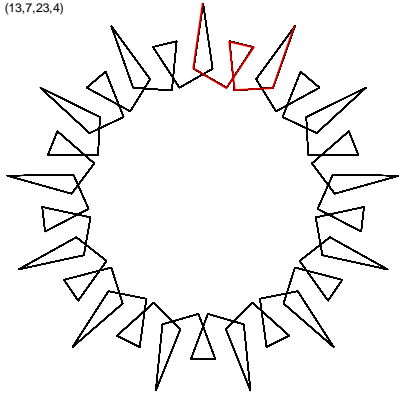
(n, S, P, J)  
(13,7,23,4)

91 lines

## Polygons and Stars in a Cycle

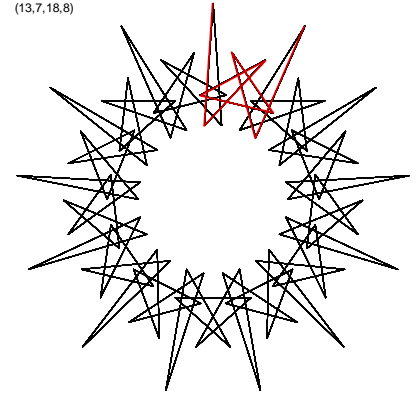
(n, S, P, J)  
(13,7,18,8)

91 lines



These one-time-around images are created when lines in each cycle connect in close proximity to one another. When viewed dynamically, these images seem to swirl around the circle like many of the 60-second images in *The Ticking Clock* explainer.

Clockwise drawn images can be turned into counterclockwise drawn ones by setting  $J$  to  $n-J$  so  $(n, S, P, J) = (13, 7, 18, 5)$  is the same as the top right image but is counterclockwise drawn.



If you find such an image, you can expand them by changing  $S$  and  $P$  for fixed  $J$  and  $n$ . The rule is simple and elegant.

- Add the smaller of  $J$  and  $n-J$  to  $S$  for the next higher  $S$  (so 7 turns into 11 at middle left or 7 to 12 at bottom left).
- Add  $n$  to  $P$  for the next higher  $P$ .

This process produces images that are similar but have one more polygon or star per cycle. Because of the overlapping and stretched nature of the polygons and stars, this can be confusing to count unless you think in terms of a "distinguished attribute" of these looped images. In the polygon version it might be the outer edge which goes from 1 in upper left to 2, 3, and 4 in the middle row from left to right (and notice that the middle edge in the odd versions have endpoints at the same level as one another). In the star version, tops are hard to see but note that the bottom points go from 2 in upper right to 3, 4, and 5 in the bottom row from left to right.

$S = 4k+3 = Jk+3$  and  $P = 13k+10 = nk+10$  given  $J = 4, n = 13$  ( $k = 1$  top left, and  $k = 2-4$  below)

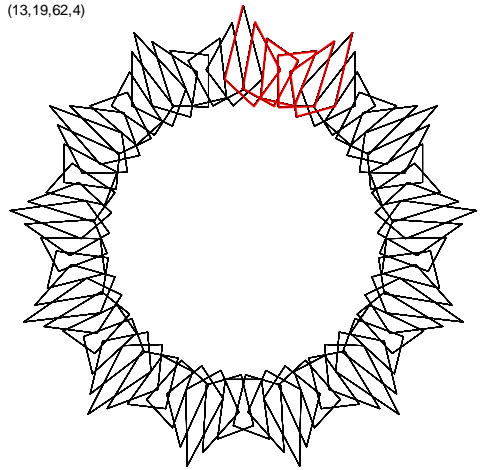
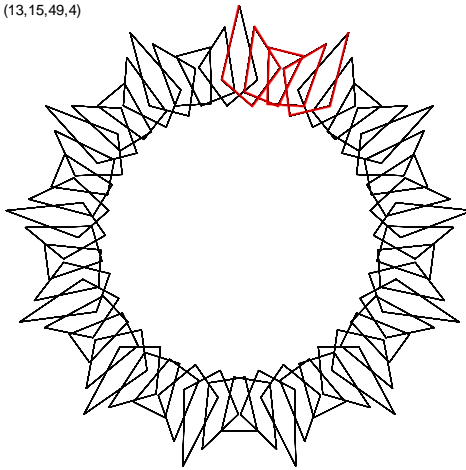
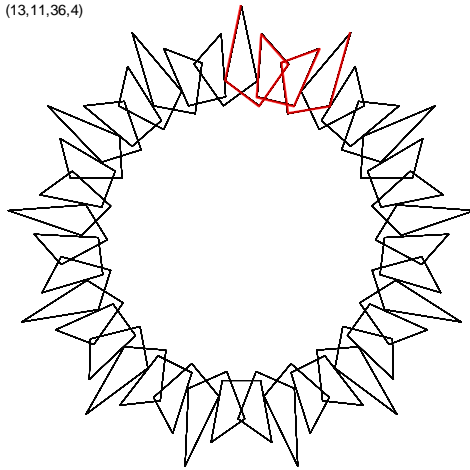
[link to middle right image](#)

(n, S, P, J)  
(13,11,36,4)

143 lines (n, S, P, J)  
(13,15,49,4)

195 lines (n, S, P, J)  
(13,19,62,4)

247 lines



$S = 5k+2 = (n-J)k+2$  and  $P = 13k+5 = nk+5$  given  $J = 8, n = 13$  ( $k = 1$  top right, and  $k = 2-4$  below)

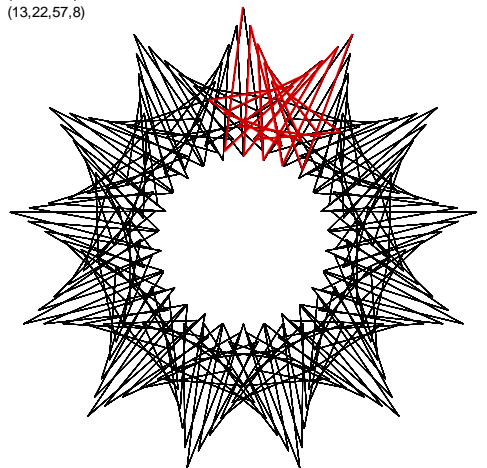
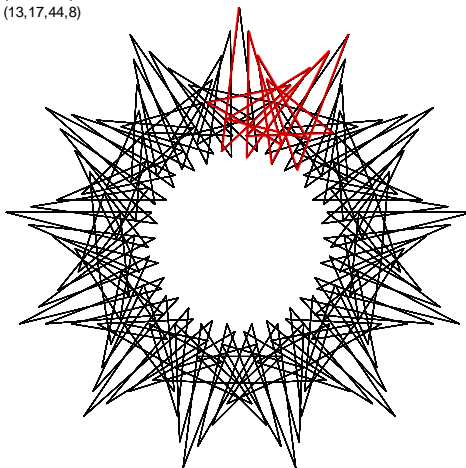
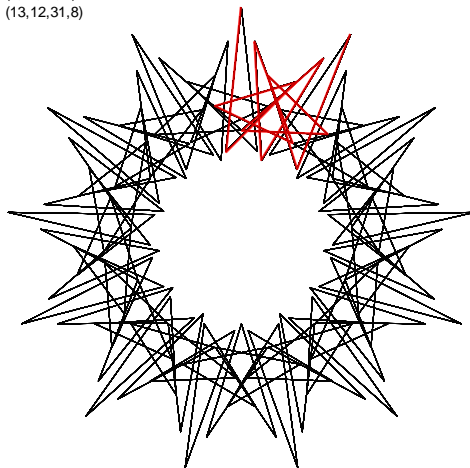
[link to bottom right image](#)

(n, S, P, J)  
(13,12,31,8)

156 lines (n, S, P, J)  
(13,17,44,8)

221 lines (n, S, P, J)  
(13,22,57,8)

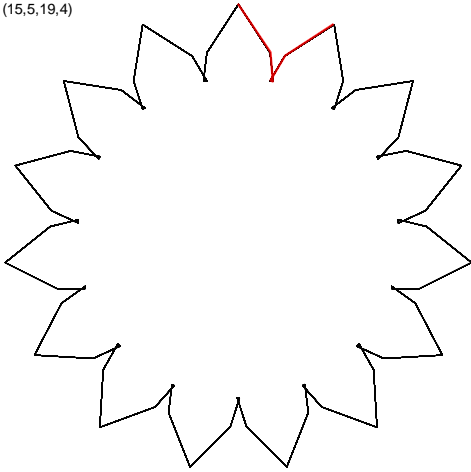
286 lines



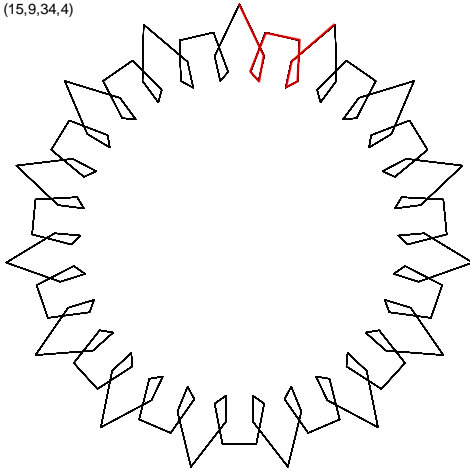
The images on the next few pages show  $k = 1-3$  for various  $J$  and  $n$ . Initial rows are shown with explicit discussion of  $S$  and  $P$  functions, but later versions leave that to the reader.

$$S = 4k+1 = Jk+1 \text{ and } P = 15k+4 = nk+4 \text{ given } J = 4, n = 15$$

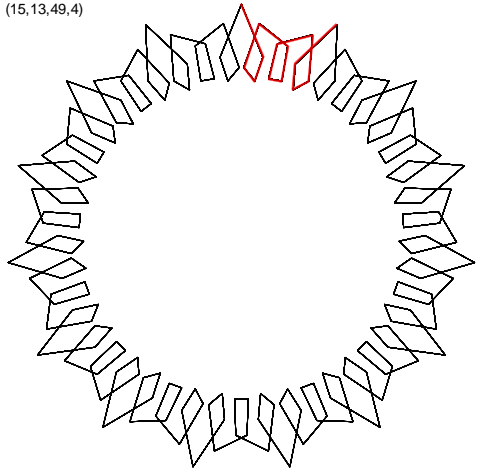
(n, S, P, J)  
(15,5,19,4)



75 lines (n, S, P, J)  
(15,9,34,4)



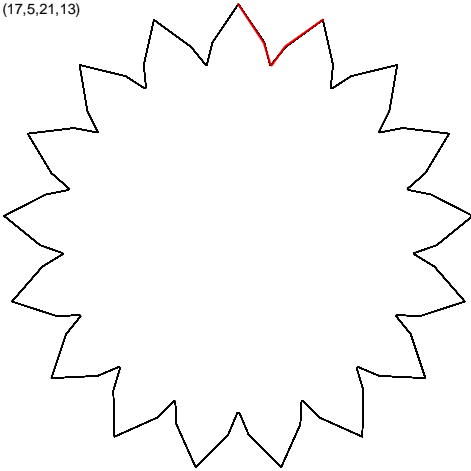
135 lines (n, S, P, J)  
(15,13,49,4)



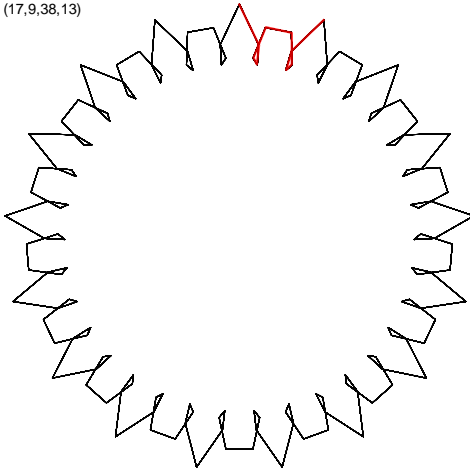
195 lines

$$S = 4k+1 = (n-J)k+1 \text{ and } P = 17k+4 = nk+4 \text{ given } J = 13, n = 17$$

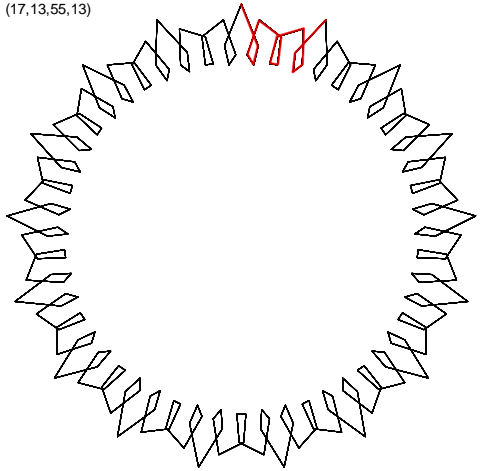
(n, S, P, J)  
(17,5,21,13)



85 lines (n, S, P, J)  
(17,9,38,13)



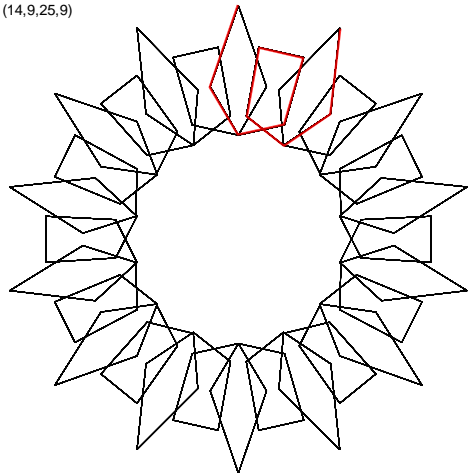
153 lines (n, S, P, J)  
(17,13,55,13)



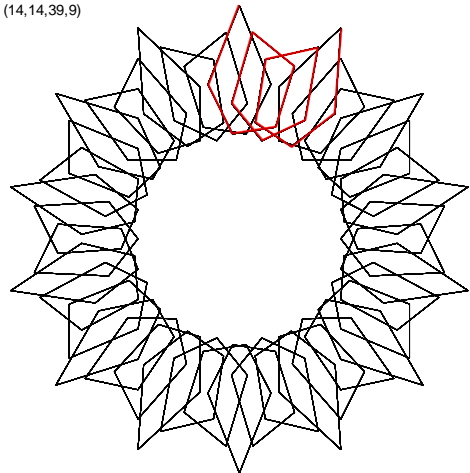
221 lines

$$S = 5k+4 = (n-J)k+4 \text{ and } P = 14k+11 = nk+11 \text{ given } J = 9, n = 14$$

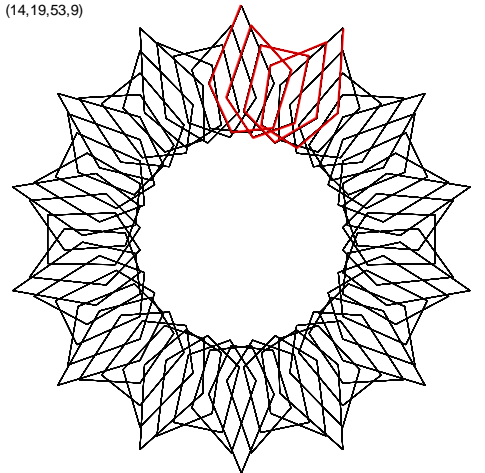
(n, S, P, J)  
(14,9,25,9)



126 lines (n, S, P, J)  
(14,14,39,9)



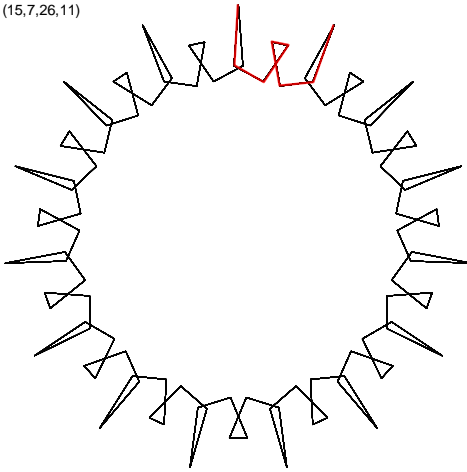
196 lines (n, S, P, J)  
(14,19,53,9)



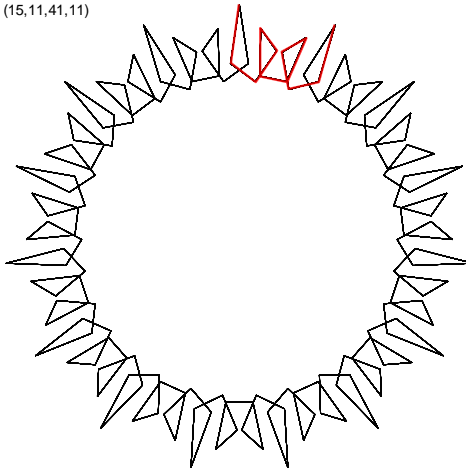
266 lines

$$S = 4k+3 = (n-J)k+3 \text{ and } P = 15k+11 = nk+11 \text{ given } J = 11, n = 15$$

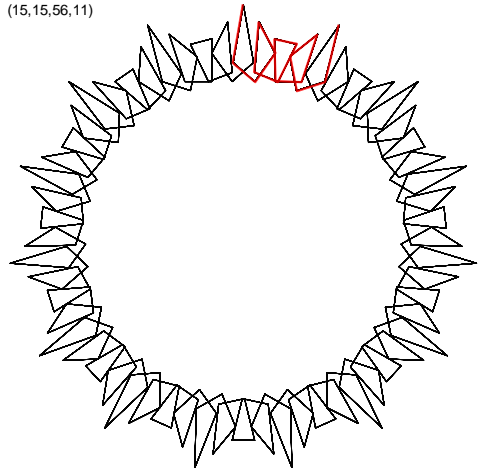
(n, S, P, J)  
(15,7,26,11)



105 lines (n, S, P, J)  
(15,11,41,11)



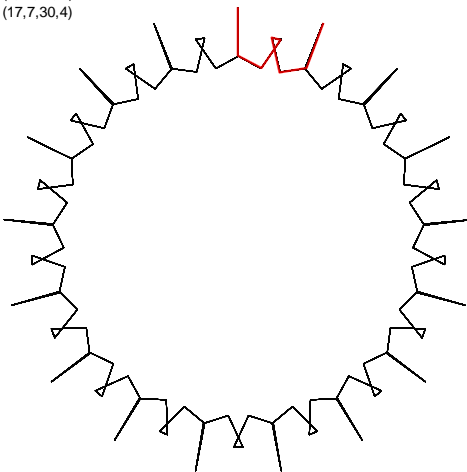
165 lines (n, S, P, J)  
(15,15,56,11)



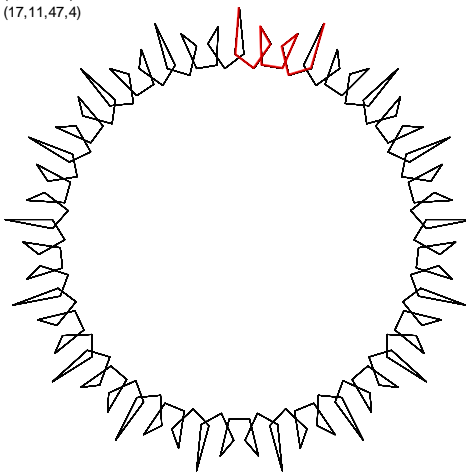
225 lines

$$S = 4k+3 = Jk+3 \text{ and } P = 17k+13 = nk+13 \text{ given } J = 4, n = 17$$

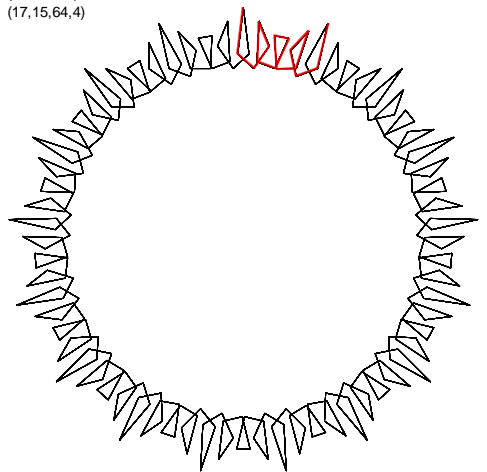
(n, S, P, J)  
(17,7,30,4)



119 lines (n, S, P, J)  
(17,11,47,4)



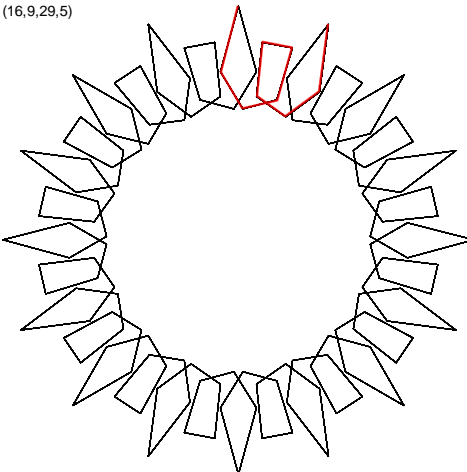
187 lines (n, S, P, J)  
(17,15,64,4)



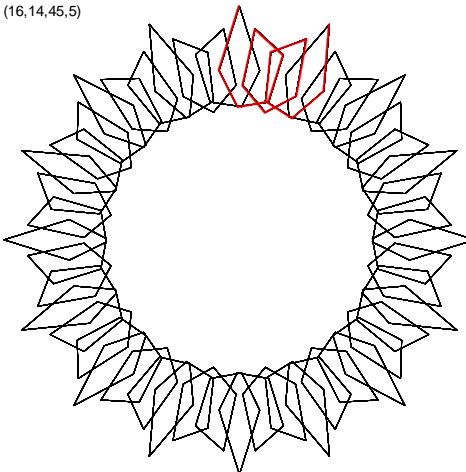
255 lines

$$S = 5k+4 = Jk+4 \text{ and } P = 16k+13 = nk+13 \text{ given } J = 5, n = 16$$

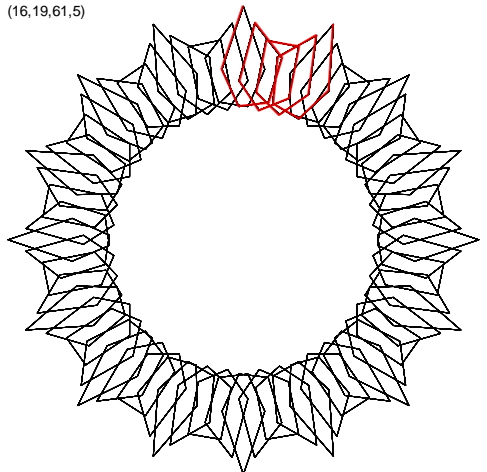
(n, S, P, J)  
(16,9,29,5)



144 lines (n, S, P, J)  
(16,14,45,5)



224 lines (n, S, P, J)  
(16,19,61,5)

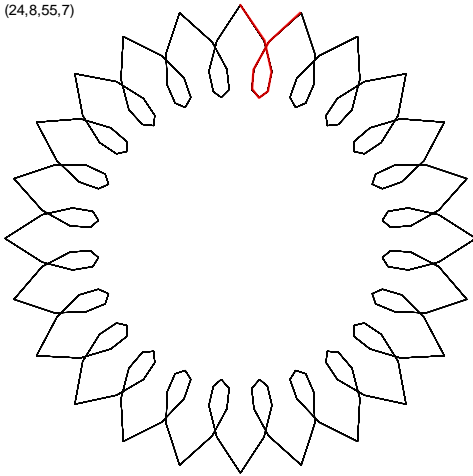


304 lines

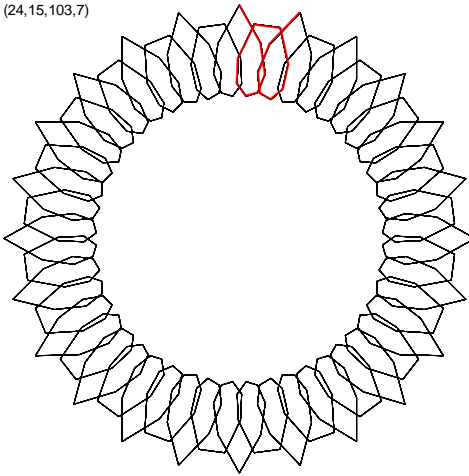
[This is a link to the image above](#)

$S = 7k+1$  and  $P = 48k+7$  given  $J = 7, n = 24$  (note that  $2n$  are added to  $P$  rather than  $n$  in this instance)

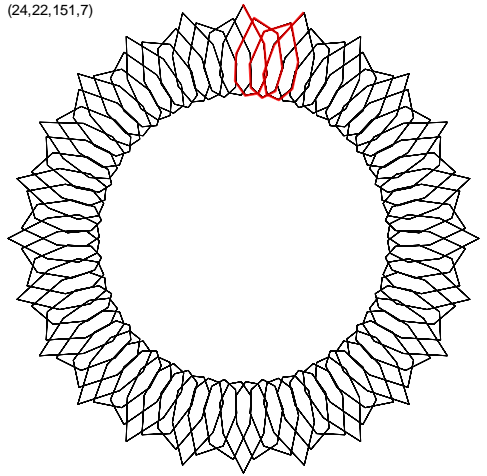
(n, S, P, J)  
(24,8,55,7)



192 lines (n, S, P, J)  
(24,15,103,7)



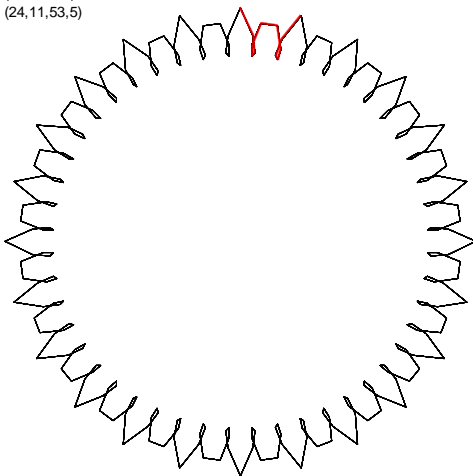
360 lines (n, S, P, J)  
(24,22,151,7)



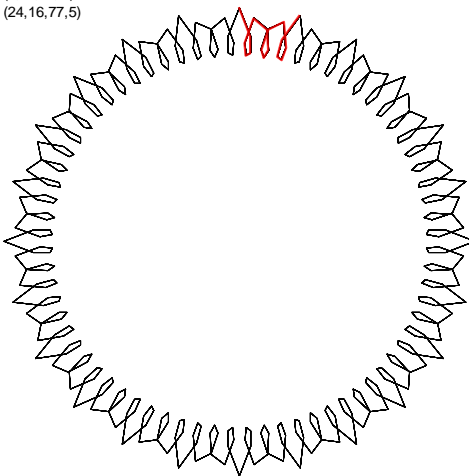
528 lines

The next row starts at  $k = 2$  (the  $k = 1$  version has a degenerate loop).

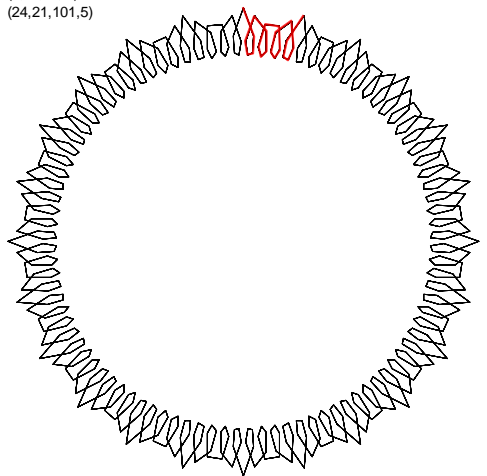
(n, S, P, J)  
(24,11,53,5)



264 lines (n, S, P, J)  
(24,16,77,5)



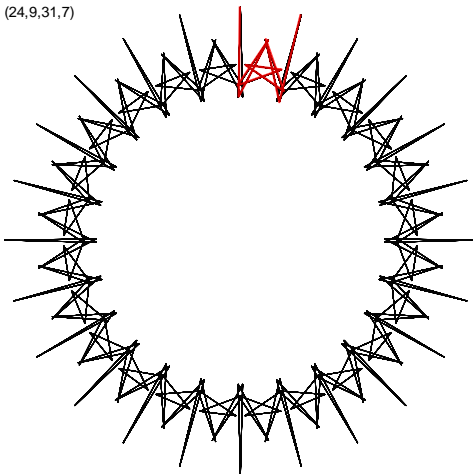
384 lines (n, S, P, J)  
(24,21,101,5)



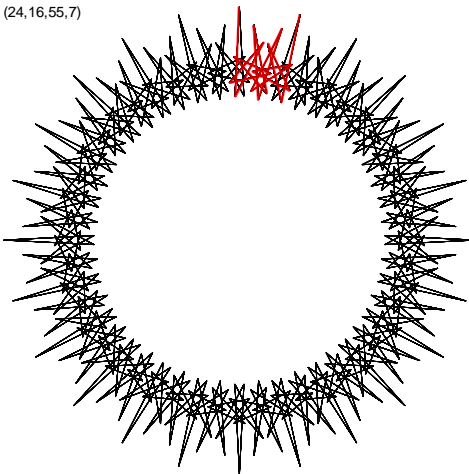
504 lines

$S = 7k+2$  and  $P = 24k+7$  given  $J = 7, n = 24$

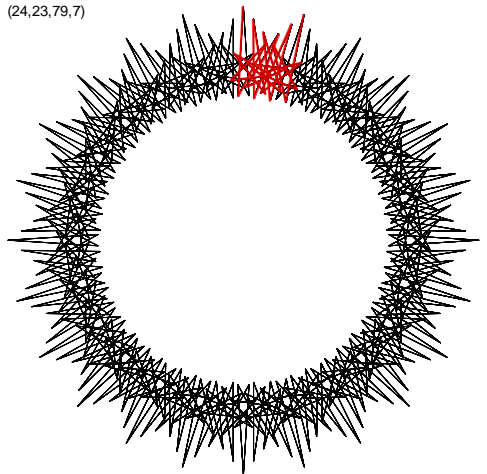
(n, S, P, J)  
(24,9,31,7)



216 lines (n, S, P, J)  
(24,16,55,7)



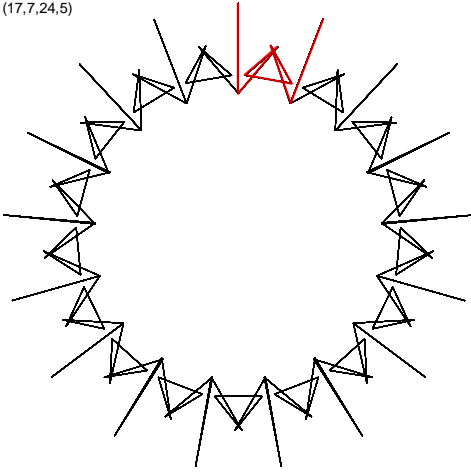
384 lines (n, S, P, J)  
(24,23,79,7)



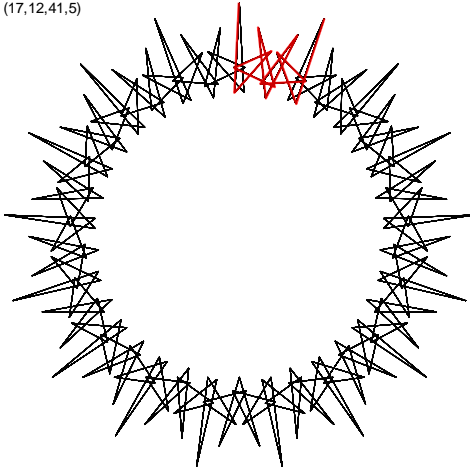
552 lines

[This is a link to the image above](#)

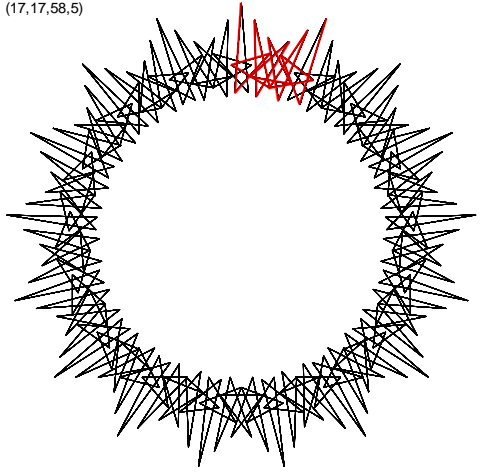
(n, S, P, J)  
(17,7,24,5)



119 lines (n, S, P, J)  
(17,12,41,5)



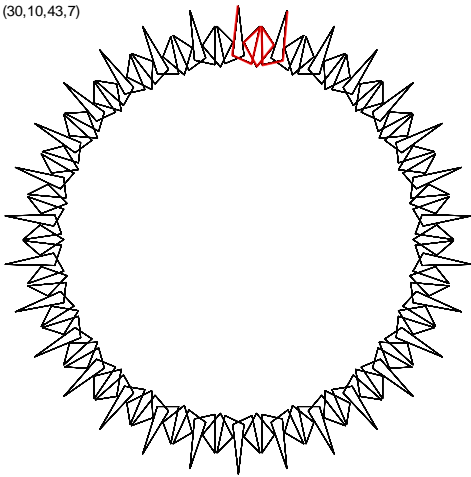
204 lines (n, S, P, J)  
(17,17,58,5)



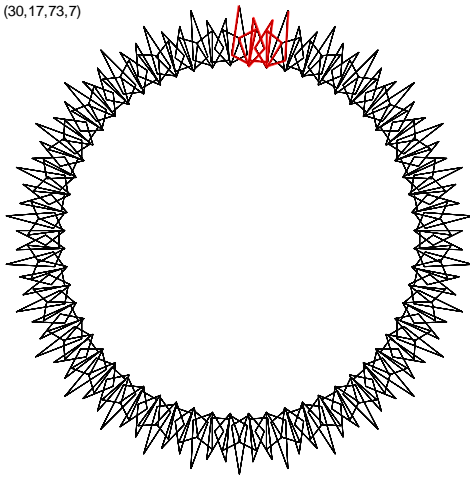
289 lines

The next two rows are based on  $n = 30$  and  $J = 7$

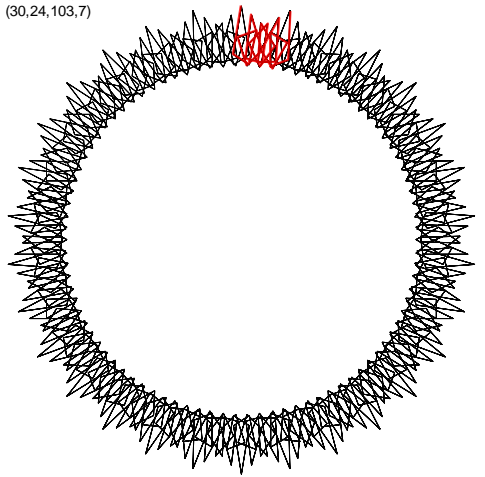
(n, S, P, J)  
(30,10,43,7)



300 lines (n, S, P, J)  
(30,17,73,7)



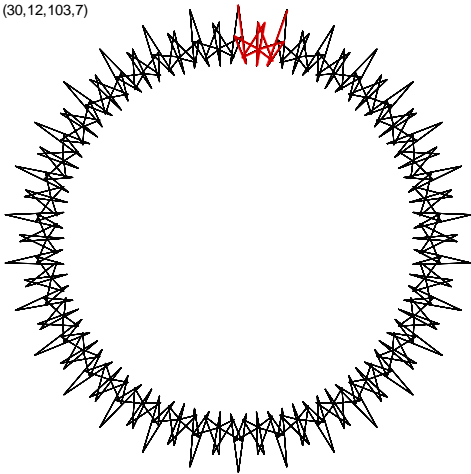
510 lines (n, S, P, J)  
(30,24,103,7)



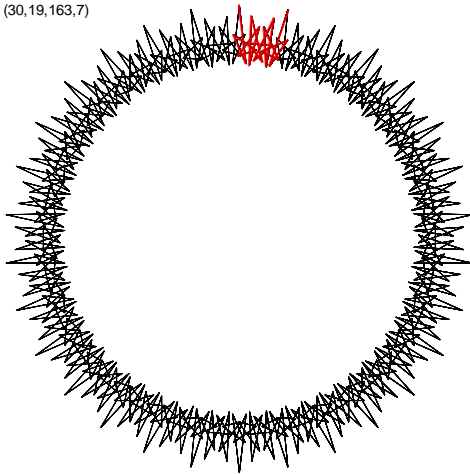
720 lines

Note the difference for the images in the row below. Following the strategy of adding  $J$  to  $S$  and  $n$  to  $P$  produces  $P = 133$ . But  $\text{GCD}(19, 133) = 19$  so the image collapses to the vertex frame. But adding  $2n$  to  $P$  produces subsequent images.

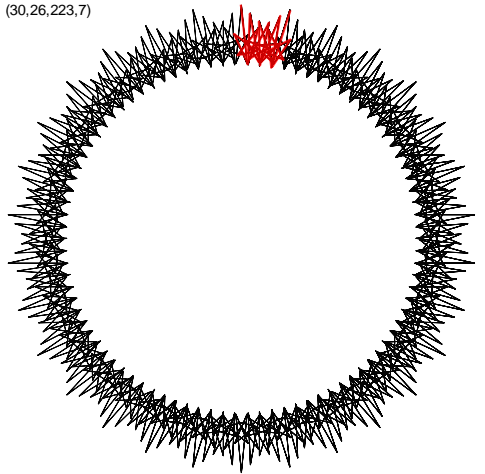
(n, S, P, J)  
(30,12,103,7)



360 lines (n, S, P, J)  
(30,19,163,7)



570 lines (n, S, P, J)  
(30,26,223,7)



780 lines

[This is a link to the image above](#)