

# Single-Step Images

If  $VCF = SCF = 1$ , then all  $nS$  subdivisions are used in the final image. Most of the time, the drawn images will get filled in in a seemingly haphazard fashion (although there is a mathematical explanation for how the image gets filled in). However, there are certain situations where adjacent subdivisions are used for sub-images of  $L$  lines long. In this instance, the image is drawn in a fashion that is quite similar to the discussion in [explainer 1.4: Stars as Rotating Polygons](#). The difference here is that now the rotating polygons morph a bit as they are based on subdivisions rather than the regular vertices of larger polygons.

The simplest versions to see are when  $L = 3$  or  $4$  and the VF is a polygon rather than a star so  $J = 1$  (or  $n-1$ ). Two versions of each are provided because  $120 = 3P = 3 \cdot 40$  on the left and  $120 = 4P = 4 \cdot 30$  on the right. The top two are “closed” because  $nS = 119$  while the bottom two are “open” because  $nS = 121$ . Here are weblinks to these four images:

[Top Left](#)

[Bottom Left](#)

[Top Right](#)

[Bottom Right](#)

(n, S, P, J)  
(7,17,40,1)

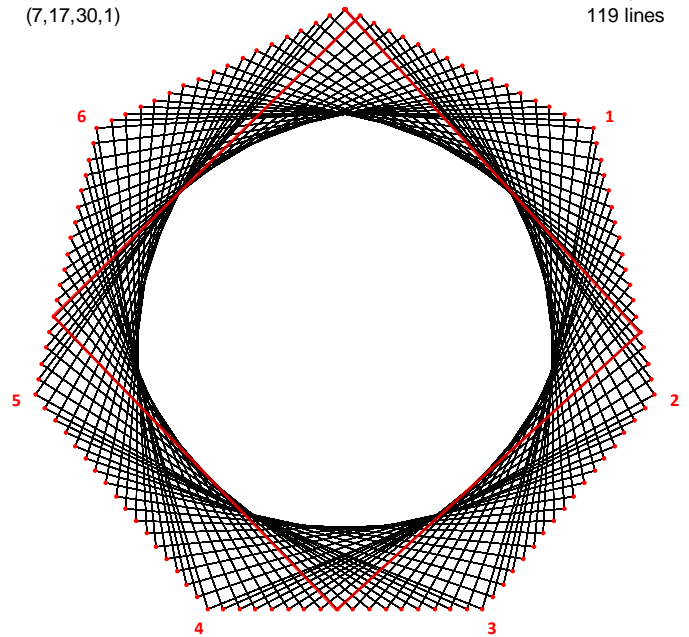
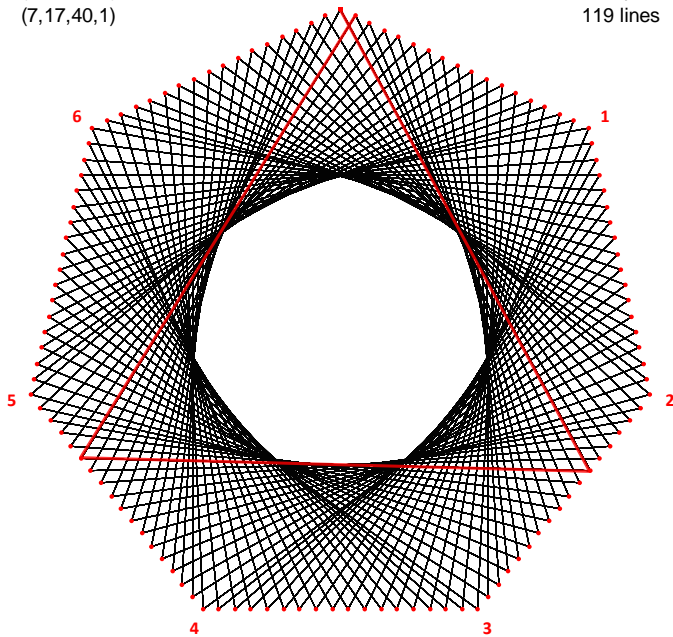
7&0

17 lines/cycle  
119 lines

(n, S, P, J)  
(7,17,30,1)

7&0

17 lines/cycle  
119 lines



(n, S, P, J)  
(11,11,40,1)

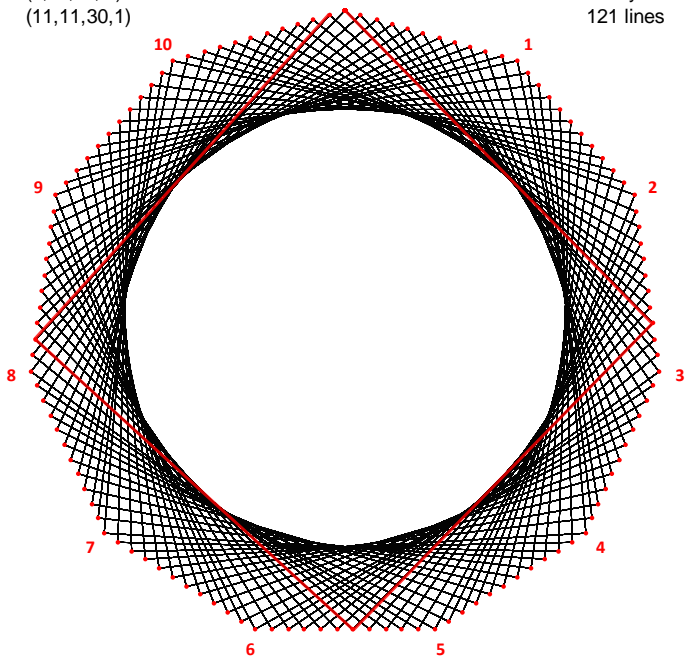
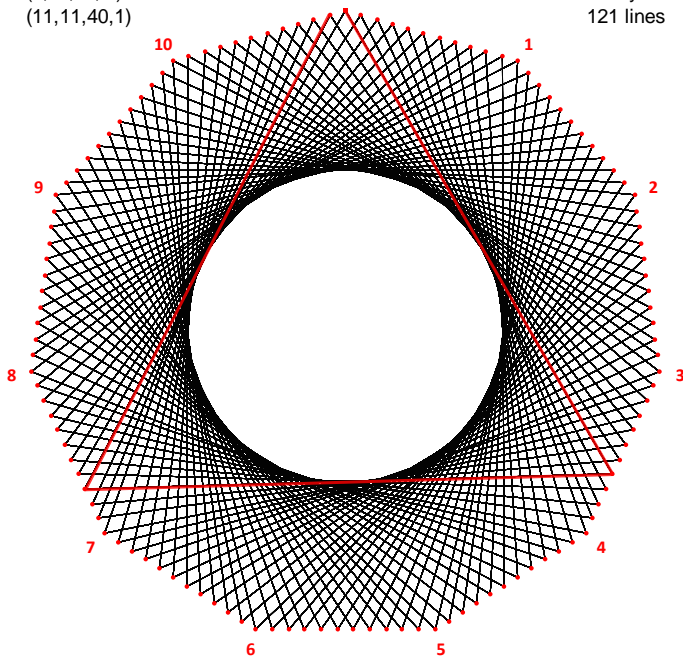
11&0

11 lines/cycle  
121 lines

(n, S, P, J)  
(11,11,30,1)

11&0

11 lines/cycle  
121 lines



**U or U?** Consider for a moment the direction of rotation of the “almost” equilateral triangles and squares implied by the end point of the first triangle or square. (To see an exact equilateral triangle or square, set  $n = 12$  and  $S = 10$  while leaving  $P = 40$  or  $30$  and  $J = 1$ .) The polygon’s endpoints are at subdivision 1 for the upper images and at  $nS-1$  for the lower images. The next set of ends will be at 2 upper and  $nS-2$  lower, and so on. **The upper images will rotate clockwise, and the lower images will rotate counterclockwise.** This is readily confirmed using the web-version’s Drawing Mode.

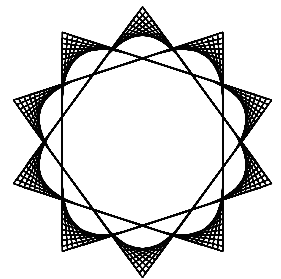
**Varying J.** Additional images with the same property are available for  $J = 2$  to 6 for the top two and  $J = 2$  to 10 for the bottom two images. Half of the images appear to rotate in a clockwise direction and the other half appear to rotate in a counterclockwise direction. The reason is simple: due to the vertical symmetry inherent to File 2 images,  $J$  and  $n-J$  produce the same static image; the only difference in images is the direction the first line is drawn (see Sections 2.3 and 2.4 explainers). It is worth noting that some of the images do not “appear” to look like triangles or rectangles, but each version maintains the single step forward or backward along the VF attribute.

**Varying L and P (for fixed n and S).** Additional  $L$  and  $P$  produce similar results so long as  $L \cdot P = 120$ . Note that  $L = 2$ ,  $P = 60$  produces porcupine polygons and stars. Additionally, in order to have the polygon have  $L$  distinct sides, one must restrict  $L$  so that  $P > S/2$  (otherwise, at least two of the sides will be collinear). For the  $S = 17$  top row, this means that  $P = 10$  (and  $L = 12$ ) is the smallest possible value of  $P$  but in the bottom row,  $P = 6$  and  $L = 20$  since  $6 > 11/2$ .

**A general rule.** A single-step image requires that there exists an  $L$  where  $\pm 1 = LP \text{ MOD } nS$ . Put another way,  $L$  and  $P$  are MMI or nMMI Mod  $nS$ . (Modular multiplicative inverses and negative modular multiplicative inverses are at the center of the [Ticking Clock](#) paper (from the end of File 2) and are further discussed in the [MMI explainer](#).)

**More complex single-step examples.** The [Three Shape-Shifting Triangles](#) image is based on the  $n = 30$ ,  $S = 19$ ,  $P = 163$ , and  $J = 13$ . This image has  $L = 7$  because  $LP = 7 \cdot 163 = 1141$ ,  $nS = 30 \cdot 19 = 570$ , so  $1 = LP \text{ MOD } nS$ .

When  $P < S$  each angle of the VF has a traditional string art curve using that angle. These curves are easiest to see when they do not overlap too much, such as  $n = 10$ ,  $S = 40$ ,  $J = 3$  (from below) with  $P = 13$  shown to the right. When  $P$  is a bit larger than  $S$  (such as 43, 47, 49, or 51) there is a traditional string art curve at each angular vertex *plus* reverse curves such as shown below. The version below with  $P = 47$  has  $L = 17$ . The 17 lines form a large open 9-point star and a smaller internal 8-point star (since  $41 \cdot 47 = 799$ ,  $-1 = 17 \cdot 47 \text{ MOD } 10 \cdot 40$ ). The final image is counterclockwise drawn in a bit fewer than 24 of these 17-line iterations ( $24 \cdot 17 = 408$ ) as can be seen using [this link](#). The version with  $P = 49$  has  $L = 49$  since  $49 \cdot 49 = 2401$  and is clockwise drawn in a bit more than 8 of these 49-line iterations ( $8 \cdot 49 = 392$ ) as can be seen using [this link](#). (Set *Drawn Lines* =  $L$  in the Drawing Mode.)

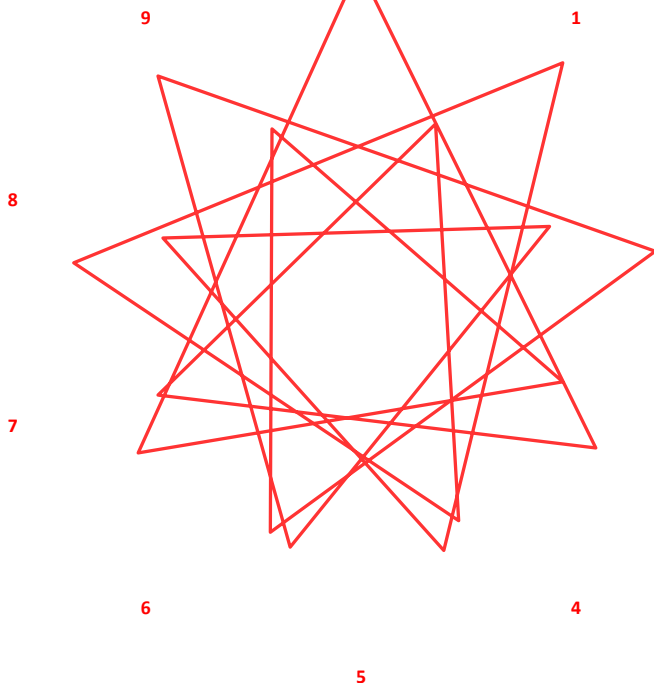


[P = 43 image](#) requires  $L = 93$  (4+ iterations) and [P = 51 image](#) requires  $L = 149$  (< 3 iterations) to be viewed as single-step.

(n, S, P, J)  
(10, 40, 47, 3)

10&0

40 lines/cycle  
400 lines (n, S, P, J)  
(10, 40, 47, 3)



40 lines/cycle  
400 lines

