

Single Cycle Images

As noted in 2.2c, each image must have AT LEAST one cycle. This explainer focuses on where there is AT MOST one as well.

At left and right are of single cycle images with n and S switching between 14 and 15. Single cycle images are created by setting P = n. We also require VCF = 1 and GCD(n,S) = 1.

Additional versions are found when P is a multiple of n, $P = k \cdot n$. Due to the symmetry of string art images we need only consider multiples $k \le S/2$. These multiples are noted in the k = 1 images by the numbers **1** to **7** (the image on the right and the versions below are cropped to save space).

k = 2

In both images, there are 210 = **nS** possible subdivision points and both are <u>One Level Change</u> images since **S** and **P** differ by only 1. Put another way, both images have full density on concentric circles of subdivision points (denoted (a) in 2.2i) but vertex usage is 1/n since only one vertex is used in the final image.

If you use the numbers from **1** to **7** in the upper images you can see why the k = 2 to 7 versions look like they do (these numbers are the ending point for the first segment of each image). Note in particular that there are 5 triangles to the left and 14-line "finger traps" to the right because GCD(k, S) = 1.

As **k** increases, we maintain full density on concentric circles unless GCD(k, S) > 1 in which case (a) density diminishes (by 1/3 for

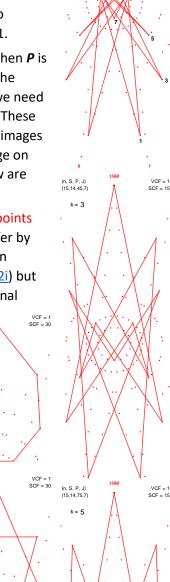
SCF = 70

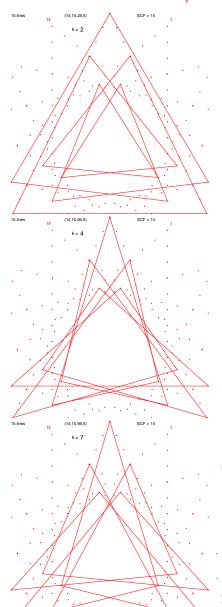
k = 3

(14, 15, 70, 5)

k = 5

k = 3 and 6 and by 1/5 for k = 5 to the left (since S = 15), and by ½ for k = 2, 4, and 6 to the right (since S = 14)). The k = 7, $P = 7 \cdot 15 = 105$ version to the right is omitted: it is the vertical line loop from vertex **15&0** to subdivision point 105 to **15&0**.





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(n, S, P, J) (15,14,15,7) VCF = 1 SCF = 15