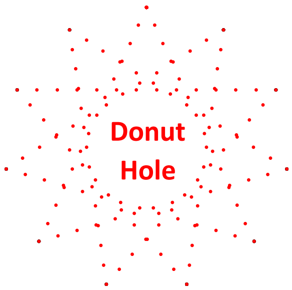
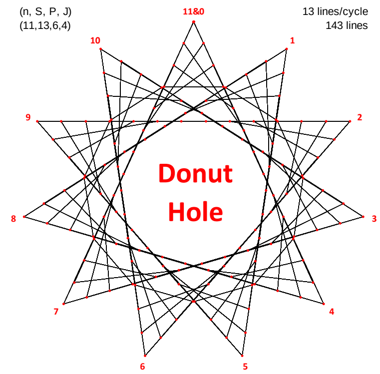
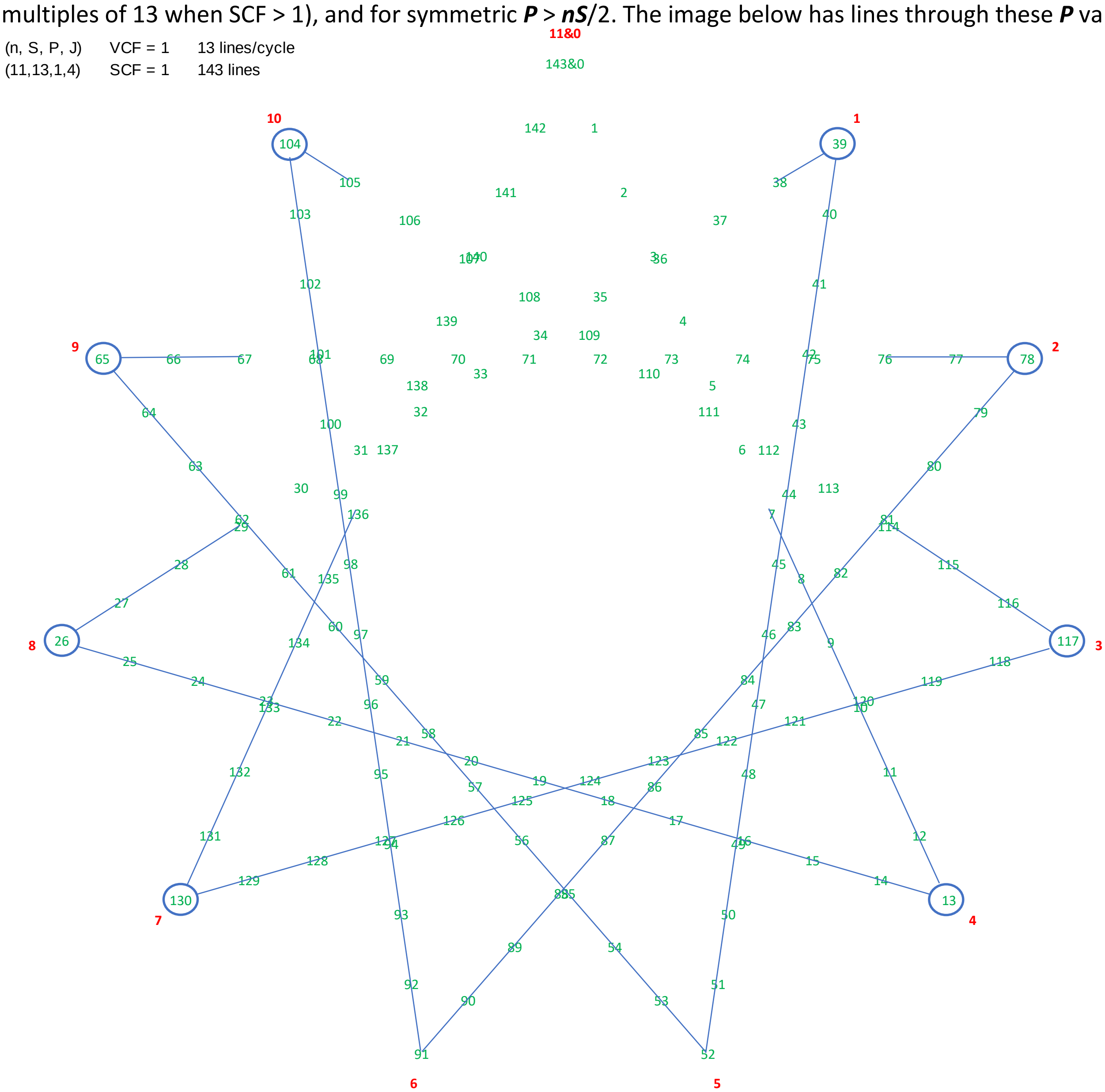


Analyzing Waves of Images and the Subdivision Donut Hole

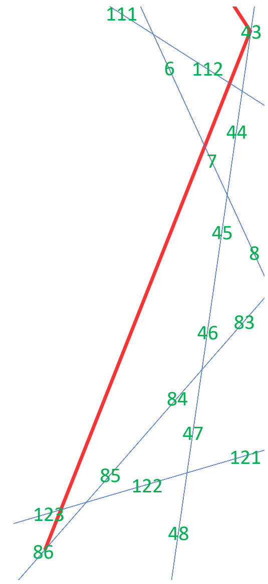
As one scrolls S or P for fixed n and J, one inevitably notices that the size of the white space in the center varies in size. Some of the time, there is no noticeable white space but at other times, the amount of white space remains reasonably fixed. Recall that subdivision points form concentric circles as noted in the [concentric circles explainer](). This "donut" of concentric circle subdivision points has a donut hole in the center.



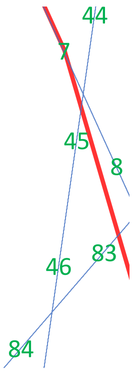
The white space does not vary in size when lines from the image are "constrained" within the band created by outer polygon and the innermost circle of subdivision points. For example, the image to the right has 6 concentric internal circles of subdivision points ($S = 2\cdot 6+1$), each has 22 points, and the outer circle of polygon vertices are the final 11 points (which is why there are $143 = 22\cdot 6+11$ subdivision points and lines). The inner white space here is an 11-gon created by the vertex frame, VF, with pointed bottom (not top like the outer polygon since J is even). $P = 6$ was chosen for this image because it is the largest $P < 30$ for which image lines do not intrude on the white space shown in the center (except $P = 13$ when the image is the VF and $P = 26$ where the image is a $J = 3$, 11-gram). The intrusion occurs once again for $37 < P < 68$ (except for multiples of 13 when SCF > 1), and for symmetric $P > nS/2$. The image below has lines through these P values.



P* values where the image encroaches on the donut hole.** Roughly speaking, ***P values in the bottom half of the image above will have lines that encroach on the donut hole. The easiest to see are ***P*** values that **inside** the angle created by polygon vertices **4-11&0-7** but are the bottom half of those concentric circles because first line (from **11&0** to ***P***) encroaches on the donut hole.



Outside this angle of subdivision points, the first line does not encroach on the donut hole, but subsequent line(s) do. For example, consider ***P*** = 43. The blow-up to the right shows the second line from **43** to **86**. Points **7**, **45**, **46**, **84**, and **85** on the border of the donut hole all lie below this line. Similar images could be created for each of the "**blue crossed-out**" subdivision numbers for at least one of the first 13 lines in the image (i.e., the first cycle).



One may wonder why the claim was made that ***P*** = 7 encroaches on the donut hole since it is hard to see that encroachment. The image to the left shows the issue. The second line segment, from **7** to **14**, encroaches just a bit on the **blue 11-gon**, cutting off the corner at the intersection of **7-8** and **44-45**.

Waves of images. Notice that encroaching images come in waves as ***P*** varies for fixed ***n***, ***S***, and ***J***. Given VCF = 1, there are ***nS*** possible subdivision endpoints and images tend to change marginally for 1 unit changes in ***P*** unless SCF > 1. As noted in explainers 2.3, 2.4a, and 2.4b, images are symmetric about ***P*** = ***nS***/2 and the same static images occur for ***J*** and ***n-J*** (but vertex numbering reverses direction) so we generally restrict out discussion to ***J*** < ***n***/2 and ***P*** < ***nS***/2.

Images encroach into the donut hole and recede from the donut hole systematically in each wave. There are 4 waves of images, one wave each that includes vertices **1**, **2**, **3**, and **4** (subdivision points **39**, **78**, **117** and **13**, respectively). More generally, there will be ***J*** waves, one each that includes part of the VF with vertices 1 to ***J*** (or ***n-J*** if ***J*** > ***n***/2) if VCF = 1.

P* values where the image does not encroach on the donut hole.** Setting ***P equal to any of the numbers that are not crossed-out in the large image on the previous page produces an image which does not extend beyond the donut of subdivisions or, to put it another way, does not encroach on the donut hole.

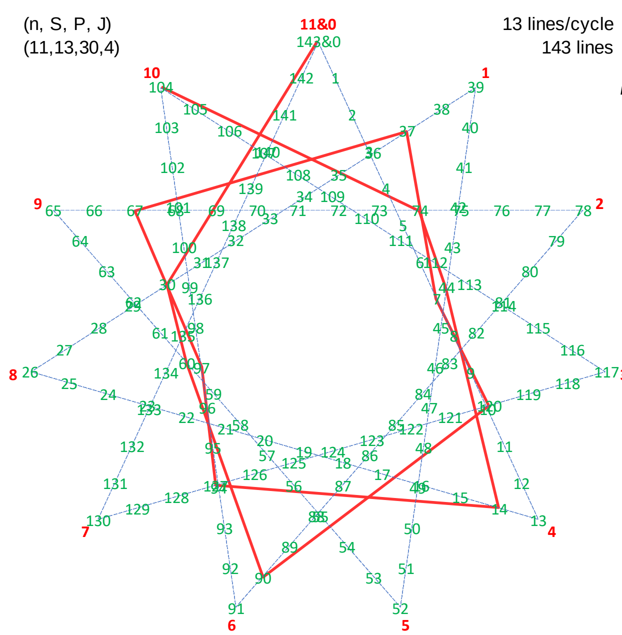
These ***P*** values are stylistically of two types: those that produce images which include the VF and those that do not. In this example, the first 6 points on the first line of the VF (***P*** < 7) and the last 6 points on the last line of the VF (***P*** > 136) include the vertex frame and are simply curved-tip stars. The rest exclude the VF.

Note that these points are generally not too far from the top of the polygon. As a result, it is instructive to look at the version that is farthest away, ***P*** = 30. The first cycle for this image is shown graphically and in tabular fashion below.

Since the first cycle ends at ***n***-1, this is a counterclockwise-drawn one-time around image (see using the Drawing Mode).

As is always the case for odd ***S*** (and SCF = 1), the cycle includes two points on each of the internal concentric circles. Note that the 5th endpoint (at **7**) and the 8th endpoint (at **97**) are on the border of the donut hole. None of the cycle's lines cross into the donut hole.

When ***S*** is even (and SCF = 1), the cycle will only have one point on the inner-most concentric circle. The 6th line of this 720-line 12-point spinning star ends on the inner-most circle (set **Drawn Lines** = 6). (This is discussed at length in The Ticking Clock).



Subdivision endpoint

k,	kP,	E_k =MOD(kP,B)
1	30	**30**
2	60	**60**
3	90	**90**
4	120	**120**
5	150	**7**
6	180	**37**
7	210	**67**
8	240	**97**
9	270	**127**
10	300	**14**
11	330	**44**
12	360	**74**
13	390	**104**