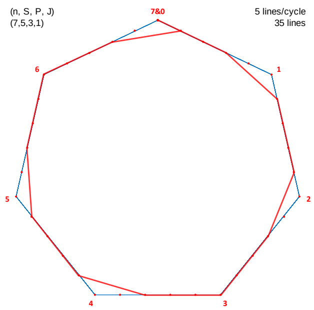
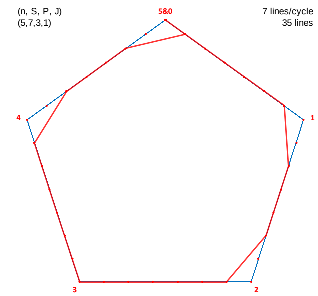
Composite Squares produce multiple *single-step* Polygons and Polygrams

One obtains single-step images of rotating ***G***-sided polygons if ***G·P*** is one more or less than ***n·S*** with ***J*** = 1. When ***J*** > 1, such images can either be polygons or polygrams. This explainer shows a trick that can be used to find multiple polygons if ***n*** and ***S*** differ by 2 and if the number in between (which we call ***C*** for center) is composite. The trick works because of a fact from algebra called the *difference between squares* formula: $(C\text{-}b)\cdot(C+b) = C^2\text{-}b^2$ (e.g., 19·21 = 399 or 18·22 = 396).

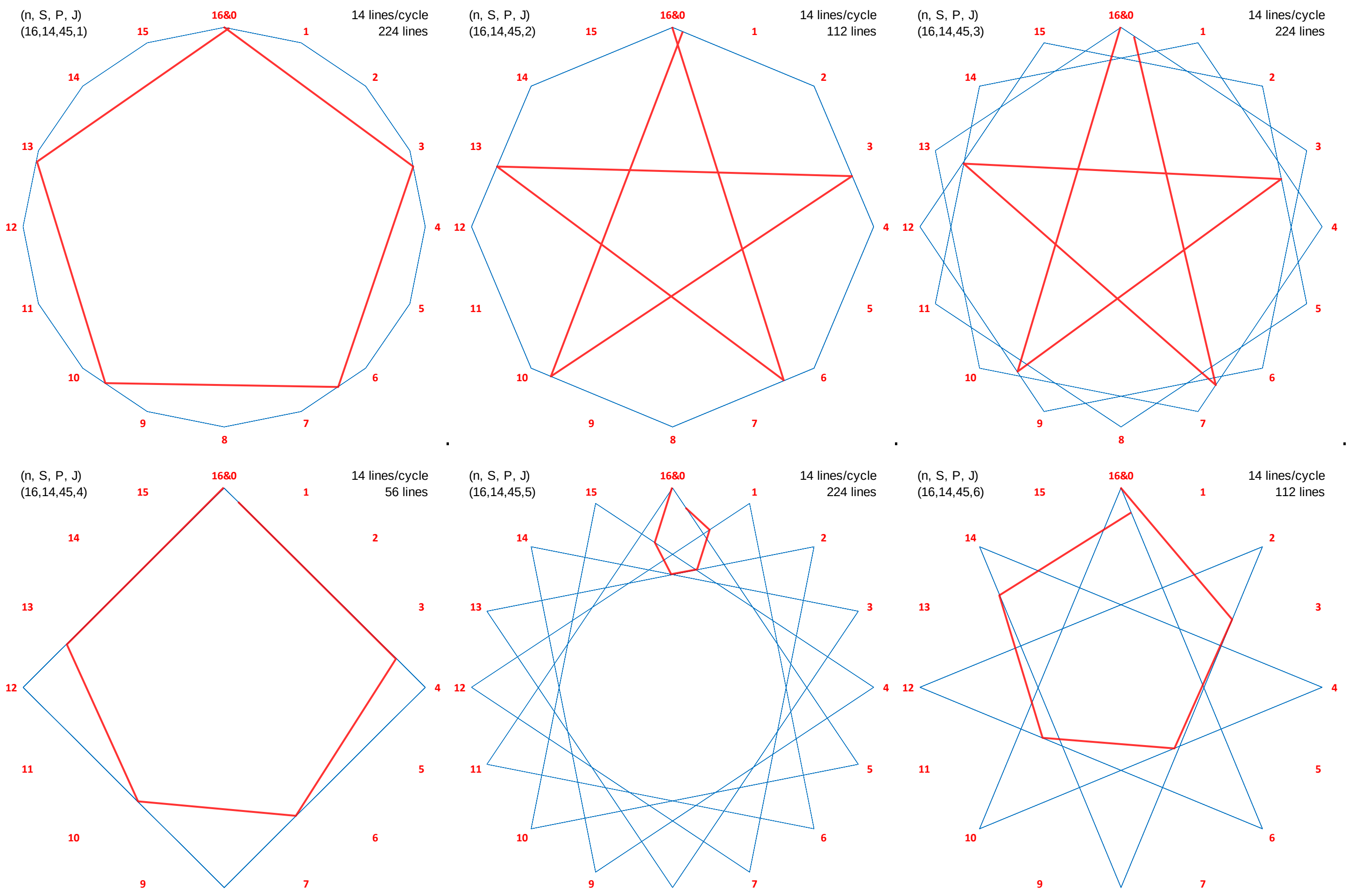
When ***b*** = 1, this formula produces a number that is 1 less than a perfect square: $(C\text{-}1)\cdot(C+1) = C^2\text{-}1$. Let ***n*** and ***S*** be the numbers on either side of ***C*** (it does not matter which is which but if you want to create as large a single-step ***G***-gon as possible, let ***S*** = ***C***-1). Each factor of C^2 (2, 3, 4, 6, 9, 12, and 18 for ***C*** = 6) can act as either the size of the ***G***-gon or the value of ***P*** necessary to produce that ***G***-gon as long as ***P*** > ***S***/2 so that successive lines have ends on different lines of the VF. Thus ***J*** = 1, ***P*** = 3, produces the 12-gon at left if ***S*** = 5 and ***n*** = 7 but only the 9-gon at right if ***S*** = 7 and ***n*** = 5 (since the 1st, 3rd and 4th VF lines have 2 segments each of the first 12 shown).





More generally, if ***G*** is a factor of C^2, ***S*** and ***n*** are the numbers on either side of ***C***, ***J*** = 1, and $P = C^2/G$ satisfies ***P*** > ***S***/2, then the resulting image is based on a clockwise rotating ***G***-gon image because $P\cdot G = C^2 = n\cdot S+1$.

Varying *J*. These images show the **first 5 lines** (***G*** = 5) and **VF** for ***J*** = 1 to 6 given ***n*** = 16, ***S*** = 14, so ***P*** = 45 = (16·14+1)/5. Similar images would have occurred had ***n*** = 14 and ***S*** = 16 been used. (Additional ***G*** are 3, 9, 15 and 25 given ***C*** = 15.)



Composite *G*. When ***G*** is composite, interesting results happen when GCD(***G***,***J***) > 1. For example, if ***G*** = ***C*** = 30, the first 30 lines are: 1 30-gon if ***J*** = 1; 2 15-gons if ***J*** = 2; 3 10-gons if ***J*** = 3; 2 15-grams if J = 4; 5 6-gons if ***J*** = 5; 6 5-gons if ***J*** = 6; 10 3-gons if ***J*** = 10; and 15 2-gons if J = 15. Switching ***n*** and ***S*** produce similar images. But, a ***G*** = 60-gon (set ***P*** = 15) is possible only if ***S*** = 29, ***n*** = 31 and ***J*** =1. In this case ***J*** = 10 is almost a 6,2 star.

C	30 = 2·3·5	***G*** -gon/ gram	2	3	4	5	6	9	10	12	15	18	20	25	30
C^2	900	$P = C^2/G$	450	300	225	180	150	100	90	75	60	50	45	36	30