## How Image Density is Distributed between Concentric Circles of Internal Subdivision Points and Polygonal Vertices



The rows switch $\boldsymbol{S}$ and $\boldsymbol{n}$ so that $\boldsymbol{n} \boldsymbol{S}=210$ and $\boldsymbol{J}$ produces the sharpest VF star possible: Top row, $\boldsymbol{n}=14=2 \cdot 7, \boldsymbol{J}=5, \boldsymbol{S}=15$; Bottom row, $\boldsymbol{n}=15=3 \cdot 5, \boldsymbol{J}=7, \boldsymbol{S}=14$. SCF varies by column from 1 to 2 to 3 to 6 by adjusting $36 \leq \boldsymbol{P} \leq 39$. As noted in 2.2i, image density depends on SCF and SCF depends on divisibility of $\boldsymbol{P}$ with $\boldsymbol{n}$ and $S$, chosen here to have no factors in common with one another in order to clarify how the interaction occurs. The table below summarizes these interactions.

| 210 lines |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | Fraction of | Reason |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dens |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $S$ used per cycle, (0) | GCD $(\boldsymbol{n}, \boldsymbol{P})$ | GCD (S, P ) |
| Top | 1 | 1 | 1 | 1 | 1 | 1/2 |  |  | 1/2 | 1 | 2 | 1 | 1/3 | 1 | 1/3 | 1 | 3 | 1/6 | 1/2 | 1/3 | 2 | 3 |
| Bottom | 1 | 1 | 1 | 1 | 1 | 1/2 | 1 | 1/2 | 1 | 2 | 1/3 | 1/3 | 1 | 3 | 1 | 1/6 | 1/3 | 1/2 | 3 | 2 |

Challenge Questions: Without looking at the String Art file, suppose $\boldsymbol{P}=77=7 \cdot 11$ or $\boldsymbol{P}=91=7 \cdot 13$. Given ( $\boldsymbol{n}, \boldsymbol{S}, \boldsymbol{J}$ ) values for Top ( $14,15,5$ ) and Bottom ( 15,14 , 7 ) rows, consider what the image might look like in each instance. In particular, what fraction of vertices and what fraction of $\boldsymbol{S}$ is used per cycle in each instance? FACT: One of these four $\boldsymbol{n}, \mathbf{S}, \boldsymbol{P}, \boldsymbol{J}$ values is a Sunburst Polygon. Can you guess as to whether $\boldsymbol{n}=14$ or $\boldsymbol{n}=15$ in this instance? Verify your answers with the file.

