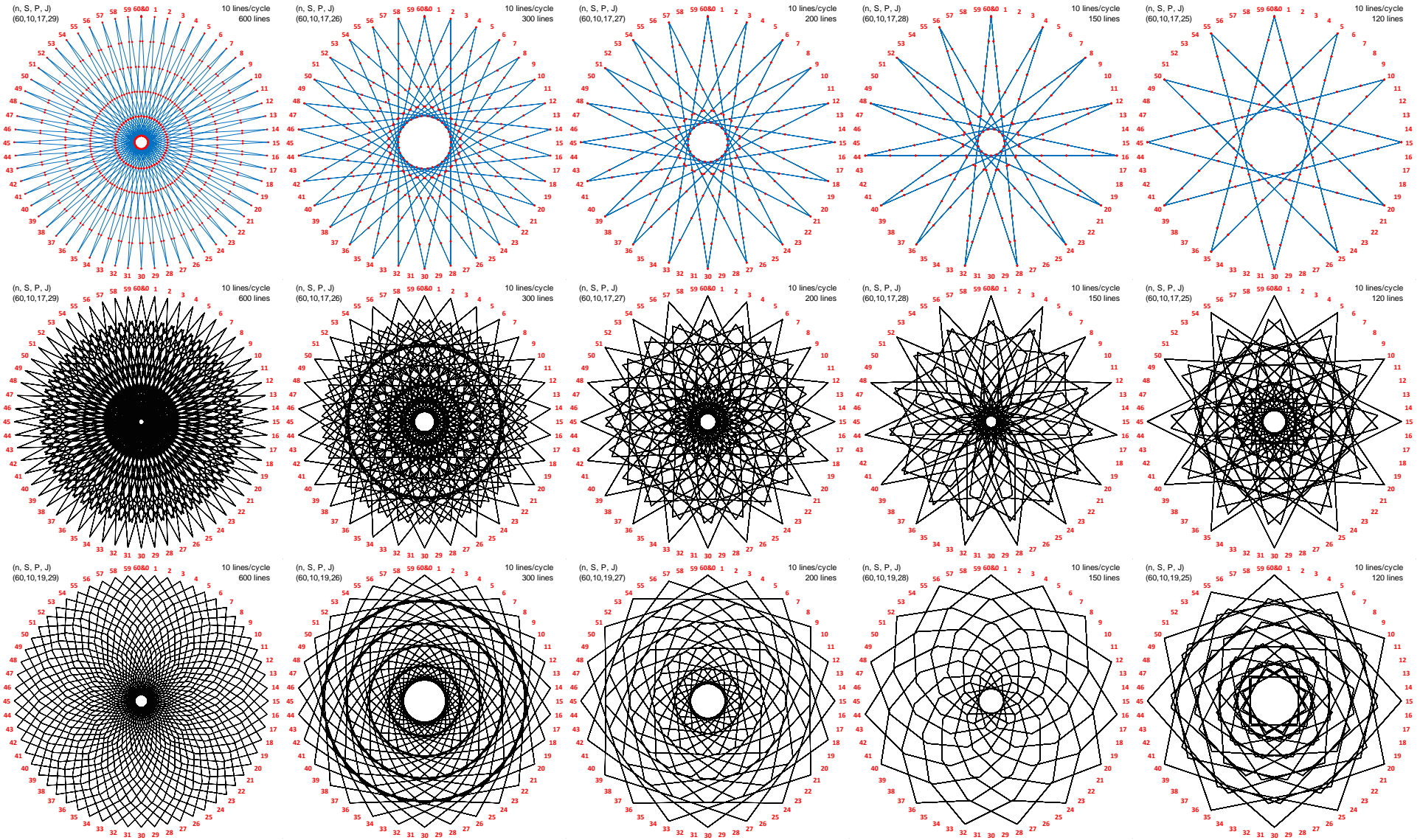


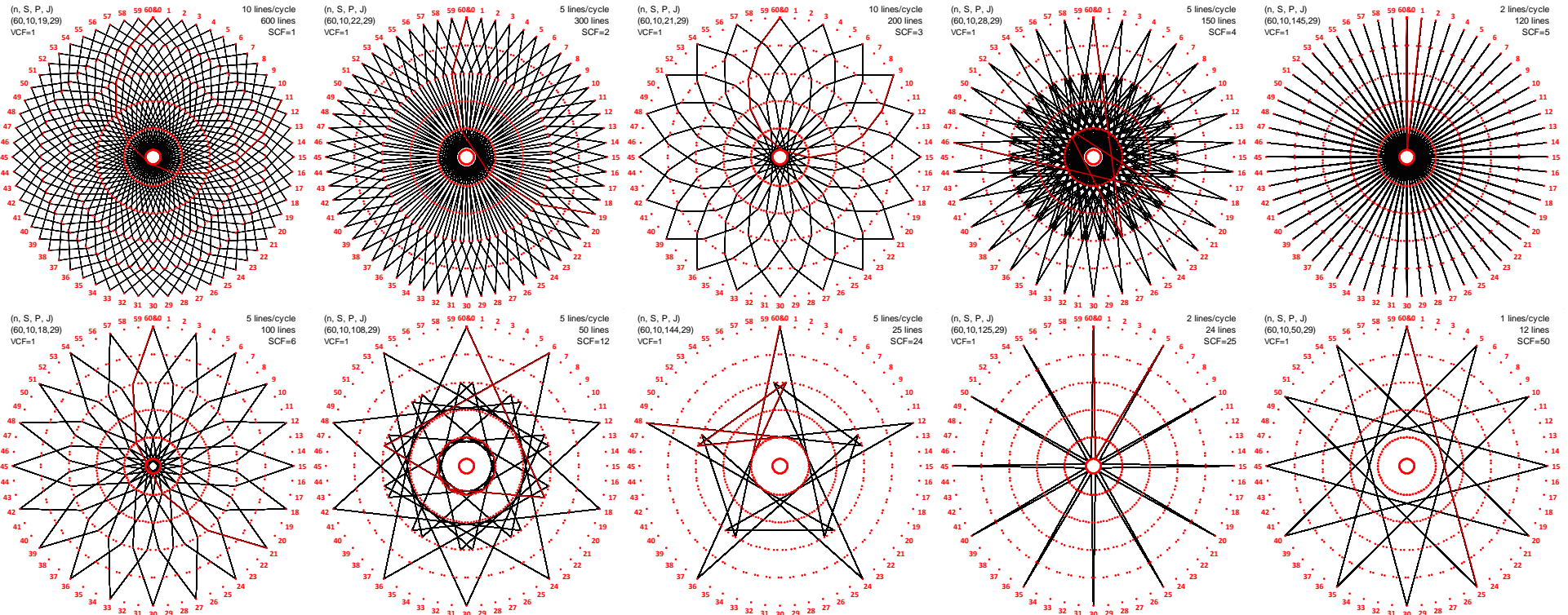
# Image Density: The Role of *Vertex Common Factor*, $VCF = GCD(n,J)$ (here $n = 60 = 2^2 \cdot 3 \cdot 5$ and $S = 10$ , $J$ varies by column)



Each column is based on  $VCF = 1$  to  $5$  from left to right based on  $J$ ,  $25 \leq J \leq 29$ . The top row shows the *vertex frame VF* (see [2.2b](#) and [2.2c](#)) and **subdivision points**, the next two rows show  $P = 17$  and  $P = 19$  images. Both  $P$  produce images with the same number of lines as **subdivision points** since  $SCF = 1$  in each case.

- 1) *Image Density* here means the number of lines in the image, relative to what is **possible**. Columns 2 to 5 have  $1/2$  to  $1/5$  as many lines as 1 since  $VCF = 2$  to  $5$ . But in a more general sense, each is full density because each **subdivision point** on the **VF** is used in the final image (because  $SCF = 1$ ).
- 2) Only vertices that are multiples of  $VCF$  are used in the **VF**. Indeed, as noted in [2.4a](#), the same image occurs if you divide  $n$  and  $J$  by  $VCF$  in each case.

# Image Density: The Role of *Subdivision Common Factor, SCF = GCD(n-S/VCF,P)* (here $n = 60, S = 10,$ and $J = 29$ so $VCF = 1$ )



The Top left image (T1) is the same as the lower left on the VCF page with the exception that, for comparison purposes, each image above shows all **600 possible subdivision points as red dots** (also on the upper left **VF** image on the VCF page) and **60 vertex labels**. Finally, the **first cycle** for each image is shown in **red**.

Recall that **Image Density** means the number of lines in the image, relative to what is **possible**. The images shown above vary **P** while holding **n, S,** and **J** fixed. **P** values were chosen so that SCF increases from 1-5 across the top row. **Image density** examines what portion of the dots are used in the final image. One can

Row (T-B)	Col. (1-5)	P	SCF	Fraction of subdivision points/Vertices used on: ◎ Vertices of n-gon	Lines per cycle = $S \cdot \text{◎}$ = $S / \text{GCD}(S, P)$	Number of cycles = $n \cdot \text{Fraction of Vertices used}$	Lines = $600 / \text{SCF} = \text{Lines/cycle} \cdot \text{\#Cycles}$
T1	19	1	1	1	10	60	600
T2	22	2	1/2	1	5	60	300
T3	21	3	1	1/3	10	20	200
T4	28	4	1/2	1/2	5	30	150
T5	145	5	1/5	1	2	60	120
B1	18	6	1/2	1/3	5	20	100
B2	108	12	1/2	1/6	5	10	50
B3	144	24	1/2	1/12	5	5	25
B4	125	25	1/5	1/5	2	12	24
B5	50	50	1/10	1/5	1	12	12

distinguish between concentric circles of subdivision points, ◎, and Vertices of the **n-gon** (columns 4 and 5 in the table). For example, half the dots are used and hence there are half as many lines in T2 because all vertices are used as are all even subdivision levels (0, 2 and 4) but odd subdivision levels (1, 3, and 5) are excluded. By contrast, T4 uses all even subdivision levels but only half the vertices so that density is  $\frac{1}{2} \cdot \frac{1}{2} = 1/4^{\text{th}}$  for T4. Similar comparisons could be made with B1-B3 and between T5 and B4.

It is worth focusing attention on T3 for a moment. This is an example of a One Level Change image (as is T1). The one third reduction in density is due entirely to the first cycle ending at vertex 9 which has  $\text{GCD}(9,60) = 3$ . But this DOES NOT MEAN that you could create this image by adjusting **n** and **J**. For example, the first subdivision endpoint used (the 21<sup>st</sup> since **P** = 21) is the 1<sup>st</sup> subdivision point on the 3<sup>rd</sup> **VF** line from **58** to **27** (since **S** = 10 and **J** = 29) and uses vertices divisible by 2 and 3 not both.

B5 uses no internal subdivisions ( $\text{GCD}(10,50) = 10$ ), so 1 in 10 points on ◎ is used,  $1/50 = 1/10 \cdot 1/5$ .