## Counting Strands and Loops in Coiled Rope Images

When you encounter a thin $\boldsymbol{n}=\boldsymbol{P}$ image that is comprised of multiple strands that are intertwined with one another, they look like a coil of rope with the ends spliced together to form a loop. Such a coil will have one or more loops and one or more strands in the rope. When the image is complex, the loops are sometimes difficult to count and the strands in the rope are also difficult to see. ( $\boldsymbol{n}=\boldsymbol{P}, \boldsymbol{S}, \boldsymbol{J}$ ) for each image is noted: upper left $(251,81,75)$, has 2 strands and 3 loops; upper right $(251,83,75)$, has 2 strands and 4 loops; lower left (499, 254, 227), has 9 strands and 3 loops; and the lower right (499, 254, 233), has 3 strands and 7 loops (and interestingly, $\boldsymbol{J}=230$ has 8 loops and $\boldsymbol{J}=236$ has 2 loops).

Loops. To count loops, start in the center and count outward avoiding intersections (so the upper right 4 loops are easily seen at the right or left side instead of top or bottom). Do not count points (so there are 3 loops in upper left not 4).
Strands. When the number of strands is difficult to see, one can use the Single Line Drawing, SLD, mode (on Pause) to good effect. For example, the lower left image has endpoint 28 just to the right of the top. Note that $254=9 \cdot 28+2$.


Watching Spirals. Rope images with multiple loops are fun to watch get drawn, or more accurately, to watch the spiral increase and decrease in size and location as the image gets created.

The top image $(499,254,233)$ is 3 smaller $\boldsymbol{S}$ than lower right image on previous page, has 3 strands and 6 loops and it shows the first 15 lines (starting at 0) in Single Line Overlaid Drawing, SLOD, mode. The $15^{\text {th }}$ line almost completes a $360^{\circ}$ loop. Smaller loops require fewer lines to complete as can be seen in the middle image, which starts at 34 . Here, the 15 line segment is about 4/3 of a loop and is about at the center of the first strand of the image (the ending of the last line before starting the second strand of the rope is at 83 (note that $83=(34+15 / 2) \cdot 2)$. Also note the number of lines in the final image, $251=3 \cdot 83+2$.

SLD mode. The best way to view this is to use SLD mode (rather than SLOD) as it does not include the static image beneath the 15 line spiral. When viewing it this way, you will note that the image curls around the smallest loop three times, once at about 34, a second time at about $117=34+83$ and the third time at about $200=117+83$.

More complex images. The bottom image $(499,121,233)$ could be described as a puffy pentagram. The rope is more loosely wound here so it is easy to see that it has 3 strands. More interesting is how many loops there are in the image. The 5 internal loops shout out to be counted, but is that all there is to the image? The SLOD mode is useful in exploring this question.

Analyze using manual steps. If you put drawing on Pause and show the first 5 lines you will see that the end of the last line is the peak of the small lower left loop.
2. Step Forward 5 clicks (so that Drawing progress shows 5) and the last line is at the top of the small upper right loop.
3. Step Forward 5 clicks (so that Drawing progress shows 10) and the last line is at the top of a large upper left loop.
4. Step Forward 5 clicks (so that Drawing progress shows 15) and the last line is at the top of the small bottom center loop.
5. Step Forward 5 clicks (so that Drawing progress shows 20) and the last line is at the top of the large upper right loop.
6. Step Forward 5 clicks (so that Drawing progress shows 25) and the last line is at the top of the small upper left loop.
7. Step Forward 5 clicks (so that Drawing progress shows 30) and the last line is near the top of the small lower right loop.
8. Step Forward 5 clicks (so that Drawing progress shows 35) and the last line is near the top of the large top center loop. The end of this line is at $40=35+5$, the end of the first strand.

We now know that this image has 8 loops ( 5 small and 3 large) as we have stepped our way through all 8 (ending at $40=8 \cdot 5$ ). Note also that total lines in the image is $121=3 \cdot 40+1$.

An exercise. Increase $\boldsymbol{S}$ by 1 to $\boldsymbol{S}=122$ (with $\boldsymbol{n}=\boldsymbol{P}=499, \boldsymbol{J}=233$ ). The 2-loop image has lots of strands. How many? Hint: SLOD.

