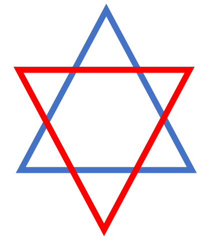
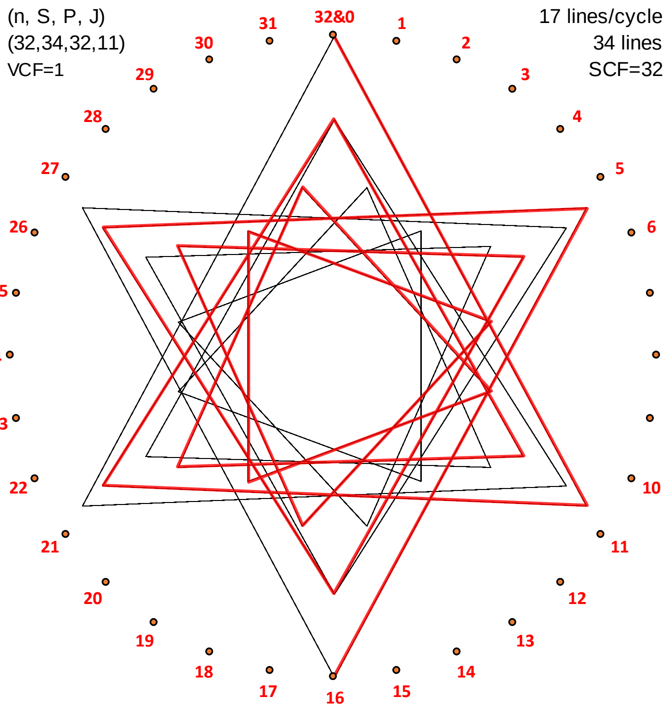
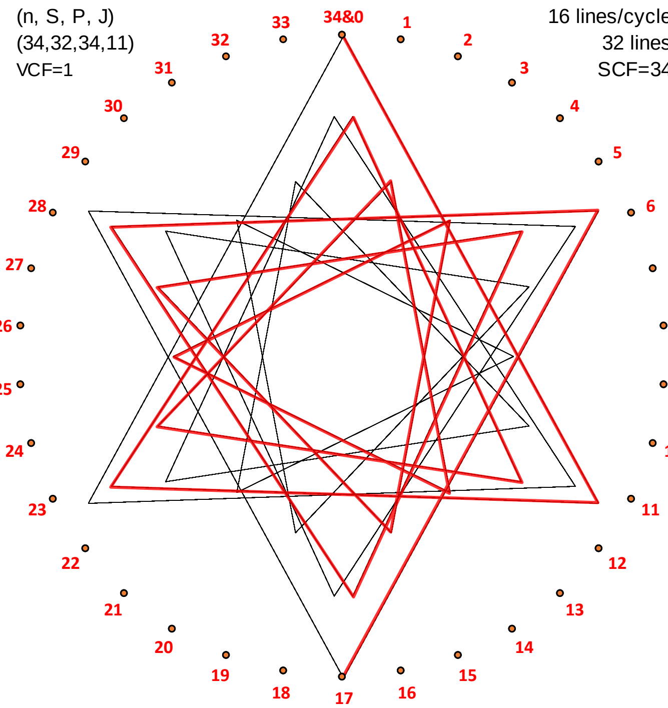
Divisibility and the Elusive 6,2 Star

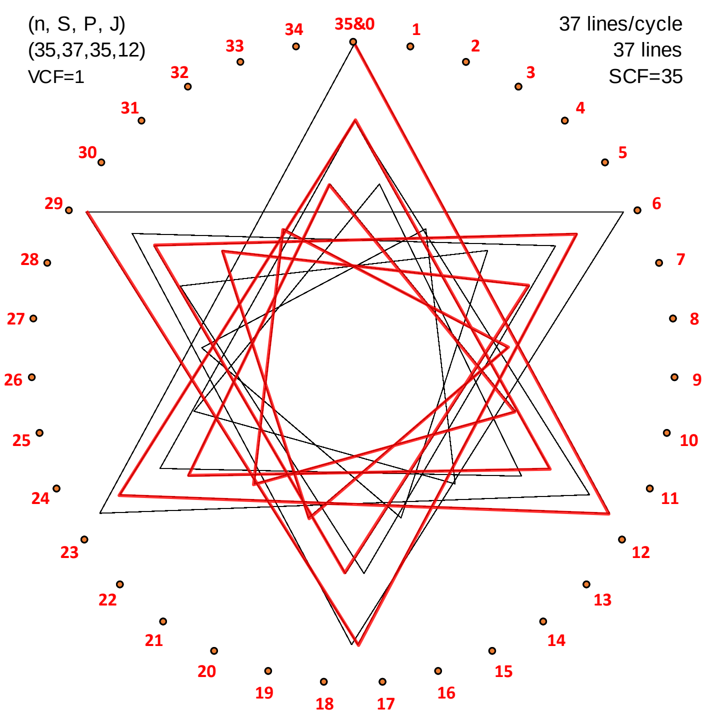


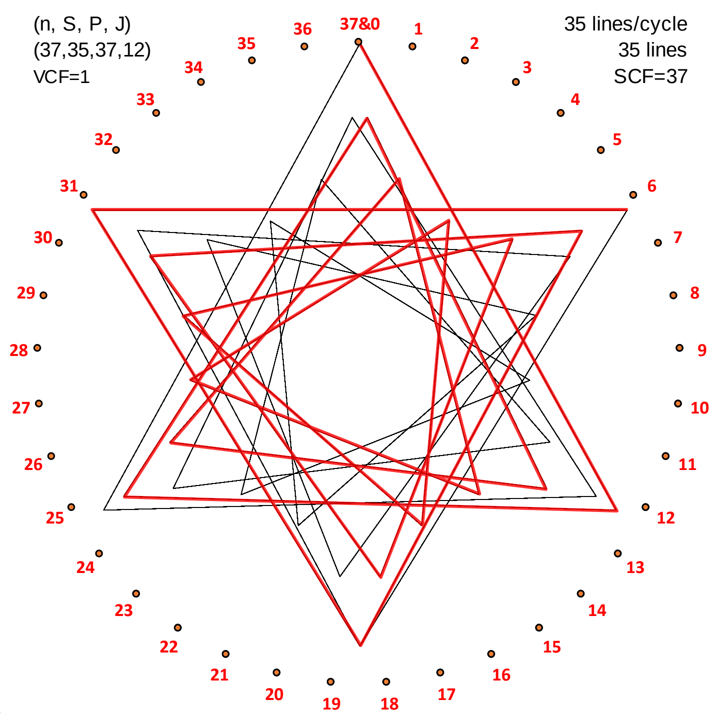
As noted in Explainer [1.2](), the only ***n*** > 4 for which a continuously drawn star cannot be drawn is ***n*** = 6. The problem in this situation is straightforward: the only number between 1 and ***n***/2 = 3 is 2 and ***J*** = 2 has a common factor with 6. The star can, of course, be drawn but not without lifting you pencil. There are 2 circuits in this instance, one **even (vertices 6&0, 2, 4)**, the other **odd (vertices 1, 3, 5)** in the image to the right. Similar issues occur whenever the *Vertex Common Factor*, VCF = GCD(***n***, ***J***) > 1 so that the only way to draw a 9,3 star is with 3 circuits, one divisible by 3 (vertices 9&0, 3, 6), one with remainder 1 upon division by 3 (vertices 1, 4, 7) and one with remainder 2 upon division by 3 (vertices 2, 5, 8).

The string art file cannot draw such stars due to the continuously drawn nature of images in the file. But if we start with a quivering triangle and adjust ***S*** up or down by 1, we can get an approximation of this image. Here are four examples.









Broad brushstrokes on these images. The four examples on the prior page exhibit the four possible types of 6,2 stars based on a 2×2 array. Each adjusts ***S*** in the opposite direction from the ***n*** = ***P*** adjustment. Consider columns and rows.

Columns: The images in the left column are based on triangles that rotate in a counterclockwise direction, ↺. This occurs when ***S*** = ***n***+2 = ***P***+2 and ***n*** = ***P*** = 3***J***–1. The images in the right column are based on triangles that rotate in a clockwise direction, ↻. This occurs when ***S*** = ***n***–2 = ***P***–2 and ***n*** = ***P*** = 3***J***+1.

Rows: The top row shows what happens when ***J*** is odd. In this instance, there are 2 cycles since ***n*** and ***S*** are even (the first of which is shown in **red** in the upper row). The bottom row shows what happens when ***J*** is even, so ***n*** and ***S*** are odd. In this instance, there is only one cycle, but the first 18 lines (6 triangles) are highlighted in **red**.

MA. Detail. This discussion leans heavily on the quivering polygons peak rotation analysis which showed why the peak triangle rotate ↻ when ***n*** = 3***J***–1 (like the left column above). When ***S*** = 3***J*** the peak returns to the top vertex after ***J*** triangles. In the current situation, the first peak (line 3) is at Level 6 instead of Level 3 for the quivering triangle counterpart. Levels change twice as rapidly here so that we are at a low Level (large triangle) at the ***J***/2nd triangle and once again at ***J***th triangle. In the context of the upper images, ***J***/2 is the 5th or 6th triangle (line 15 or 18). The upper left ***J***/2 peaks are near vertices **26** and **27** at Levels 4 and 2 respectively according to the first line placement table below, an approximate 1/6th rotation ↺. The upper right ***J***/2nd peaks are near vertices **6** and **5** at Levels 2 and 4 according to the second table below, an approximate 1/6th rotation ↻. In each instance, the second half of the triangles continue rotating in the same direction ending at approximately 1/3rd of a rotation. The bottom image rotations are easy to see because the 6th peak (line 18) is the end of the red line. Both are at Level 1, the left is near vertex **29** ↺, and the right is near **6** ↻.

Line *k*	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
Sub End	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	0	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	0
Segment	0.94	1.882	2.824	3.765	4.706	5.647	6.588	7.529	8.471	9.412	10.35	11.29	12.24	13.18	14.12	15.1	16.0	16.9	17.9	18.8	19.8	20.7	21.6	22.6	23.5	24.5	25.4	26.4	27.3	28.2	29.2	30.1	31.1	32.0
VF Start	0	11	22	1	12	23	2	13	24	3	14	25	4	15	26	5	16	16	27	6	17	28	7	18	29	8	19	30	9	20	31	10	21	0
VF Stop	11	22	1	12	23	2	13	24	3	14	25	4	15	26	5	16	27	27	6	17	28	7	18	29	8	19	30	9	20	31	10	21	0	11
Level	2	4	6	8	10	12	14	16	16	14	12	10	8	6	4	2	0	2	4	6	8	10	12	14	16	16	14	12	10	8	6	4	2	0
ΔLevel	2	2	2	2	2	2	2	2	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0	2	2	2	2	2	2	2	2

The Vertex Frame (VF) start and stop values are the **RED** numbered vertices of the polygon.

Line *k*	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
Sub End	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	0
Segment	1.06	2.125	3.188	4.25	5.313	6.375	7.438	8.5	9.563	10.63	11.69	12.75	13.81	14.88	15.94	17.0	18.1	19.1	20.2	21.3	22.3	23.4	24.4	25.5	26.6	27.6	28.7	29.8	30.8	31.9	32.9	34.0
VF Start	11	22	33	10	21	32	9	20	31	8	19	30	7	18	29	17	28	5	16	27	4	15	26	3	14	25	2	13	24	1	12	0
VF Stop	22	33	10	21	32	9	20	31	8	19	30	7	18	29	6	28	5	16	27	4	15	26	3	14	25	2	13	24	1	12	23	11
Level	2	4	6	8	10	12	14	16	14	12	10	8	6	4	2	0	2	4	6	8	10	12	14	16	14	12	10	8	6	4	2	0
ΔLevel	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

The Vertex Frame (VF) start and stop values are the **RED** numbered vertices of the polygon.

Line *k*	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
Sub End	35	33	31	29	27	25	23	21	19	17	15	13	11	9	7	5	3	1	36	34	32	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	0
Segment	0.95	1.892	2.838	3.784	4.73	5.676	6.622	7.568	8.514	9.459	10.41	11.35	12.3	13.24	14.19	15.1	16.1	17.0	18.0	18.9	19.9	20.8	21.8	22.7	23.6	24.6	25.5	26.5	27.4	28.4	29.3	30.3	31.2	32.2	33.1	34.1	35.0
VF Start	0	12	24	1	13	25	2	14	26	3	15	27	4	16	28	5	17	29	29	6	18	30	7	19	31	8	20	32	9	21	33	10	22	34	11	23	0
VF Stop	12	24	1	13	25	2	14	26	3	15	27	4	16	28	5	17	29	6	6	18	30	7	19	31	8	20	32	9	21	33	10	22	34	11	23	0	12
Level	2	4	6	8	10	12	14	16	18	17	15	13	11	9	7	5	3	1	1	3	5	7	9	11	13	15	17	18	16	14	12	10	8	6	4	2	0
ΔLevel	2	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	0	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	2

The Vertex Frame (VF) start and stop values are the **RED** numbered vertices of the polygon.

Line *k*	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Sub End	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	0
Segment	1.06	2.114	3.171	4.229	5.286	6.343	7.4	8.457	9.514	10.57	11.63	12.69	13.74	14.8	15.86	16.9	18.0	19.0	20.1	21.1	22.2	23.3	24.3	25.4	26.4	27.5	28.5	29.6	30.7	31.7	32.8	33.8	34.9	35.9	37.0
VF Start	12	24	36	11	23	35	10	22	34	9	21	33	8	20	32	7	19	6	18	30	5	17	29	4	16	28	3	15	27	2	14	26	1	13	0
VF Stop	24	36	11	23	35	10	22	34	9	21	33	8	20	32	7	19	31	18	30	5	17	29	4	16	28	3	15	27	2	14	26	1	13	25	12
Level	2	4	6	8	10	12	14	16	17	15	13	11	9	7	5	3	1	1	3	5	7	9	11	13	15	17	16	14	12	10	8	6	4	2	0
ΔLevel	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	0	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2

The Vertex Frame (VF) start and stop values are the **RED** numbered vertices of the polygon.

Creating divisible stars. The starting point for creating "close to" divisible images such as a 6,2 star, is to start with a smaller size quivering polygon. Use a triangle for the 6,2 star; a pentagon for a 10,2 star; a quadrangle for an 8,2 star or a triangle for a 9,3 star. Consider 12-point stars. All but 12,5 are divisible and are not possible to create as exact images. Close to images are possible: use a triangle for a 12,4 star, a quadrangle for a 12,3 star and a hexagon for a 12,2 star.

Once you decide on the type of polygon (with ***G*** sides) you must decide how many of those polygons are used to create the final image. The number of polygons is ***J***. From here we set ***n*** = ***P*** = ***G·J*** ± 1. Set ***S*** = ***G·J*** ∓ 1. Notice that the directions of adjustment switch between ***n*** = ***P*** and ***S***. The resulting image will be a 2***G*** star. If you want a 3***G*** or 4***G*** star, the difference |***n*** – ***S***| must be ***S*** = ***G·J*** ∓ 2 or 3.

An exercise. By adjusting ***S*** or ***J*** turn this (299,301,299,100) 6,2 star into a 9,3 star. Would you change **J** or ***S*** and to what? Turn the 9,3 star into a 12,4 star. Start again with the 6,2 star. Turn it into an 8,2 star, a 10,2 star, and a 12,2 star.