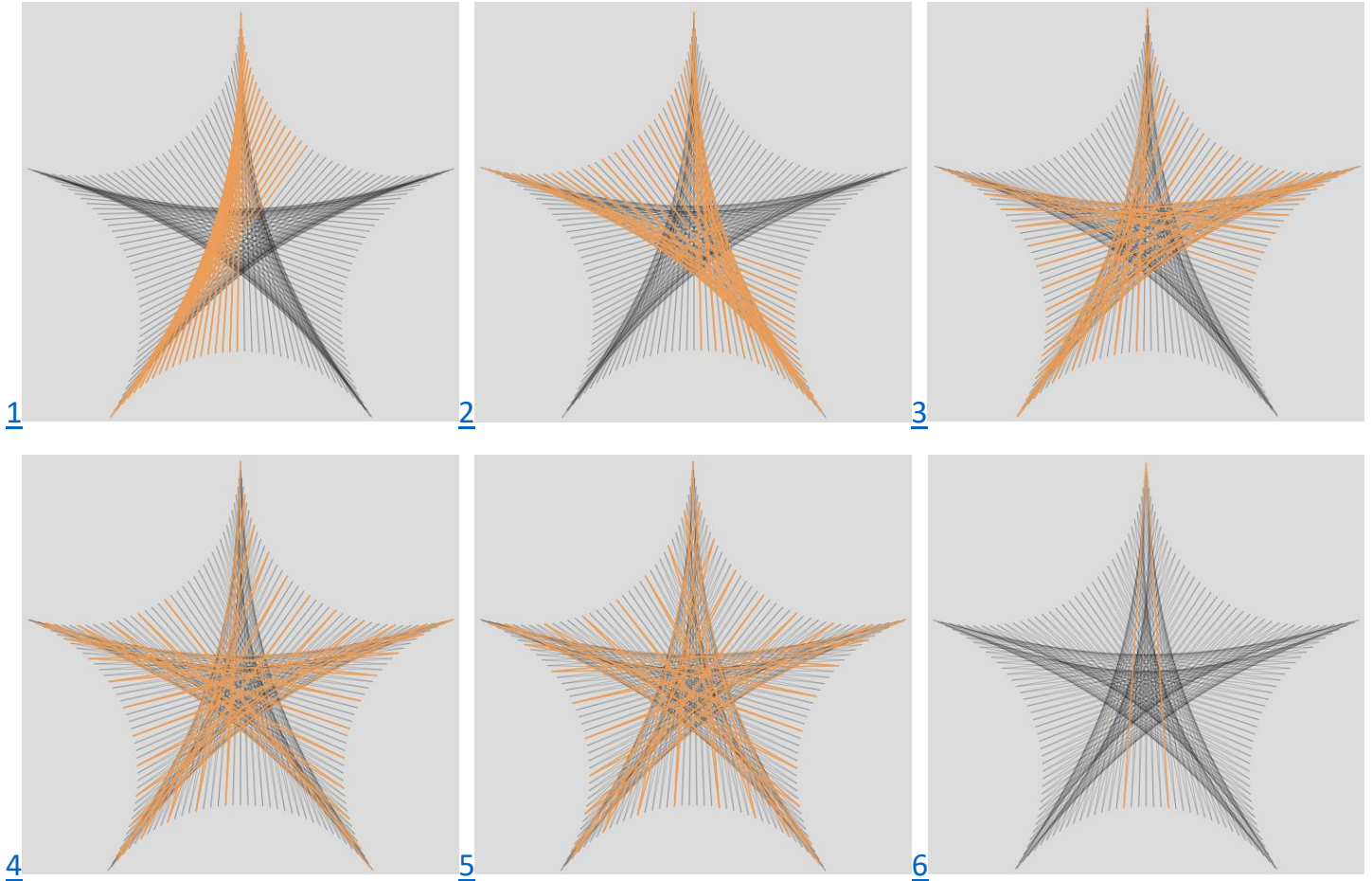


On the Distance between Paired Suspension Curves

Often in $n = P$ images one finds two curves seemingly parallel to one another anchored at each end by the same point. Two analogies are power or (old school) telephone lines between poles or the cables that suspend suspension bridges but the analogy I prefer to work with here is a pair of chains between fixed endpoints. (Interestingly, a quick *Google* search shows that the curves created in these situations (catenary versus parabolic) differ from one another.)

My interest here is in simply describing the distance between the curves using the tools we have at hand. Consider a series of 5-point webbed stars created via the equations: $S = 2J - a$ and $n = P = 3J + a$. To avoid divisibility issues, set $J = 101$ since 101 is prime. We initially vary a from 1 to 6. The first 5 highlight the first 1/5th of the image (40 or 39 lines).



The last image, $a = 12$, highlights 2 lines, *Drawing Progress* = 37.

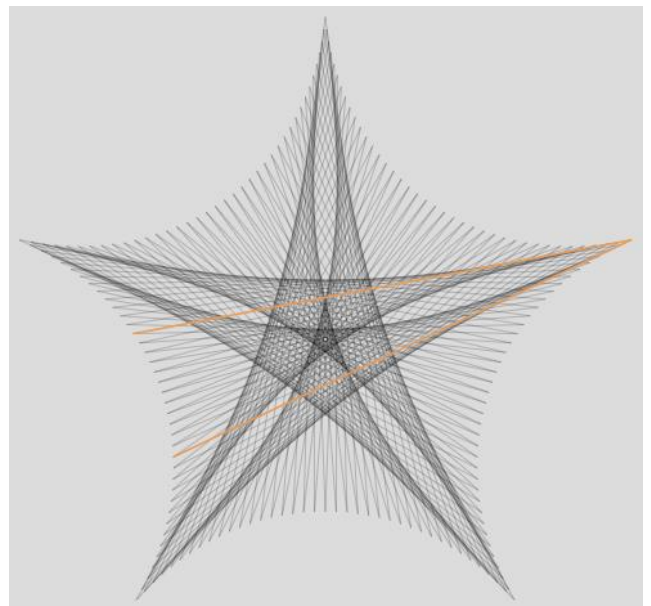
Each image has two sets of [line curves](#) spanning two of the five points as well as point curves between successive points of the 5-point star. Our focus is on the line curves (hereafter curves).

Density of curve creation. As a increases, so does the distance between adjacent used points on the point curves.

The $a = 1$ image is [smallest step](#) and is created in a single pass.

Both 0-3 curves are fully created in the first 5th of lines from lines ending in the first half of the 0-1 point curve and last half of the 2-3 point curve.

The $a = 2$ image is second smallest step and is created in two passes around the star. The 0-2 and 2-4 curves are created at 50% density in the first 1/5th of lines because every other used line endpoint is used the first time around.



The $\alpha = 3$ image is 3rd smallest step and is created in three passes around the star. The 0-3, 3-1, and 1-4 paired curves are created at 33% density in the first 1/5th of lines because every 3rd used line endpoint is used the first time around.

The $\alpha = 4$ image is 4th smallest step and is created in four passes around the star. The 0-3, 3-1, 1-4 and 4-2 curves are created at 25% density in the first 1/5th of lines because every 4th used line endpoint is used the first time around.

The $\alpha = 5$ image is 5th smallest step and is created in five passes around the star. All five pairs of curves are created at 20% density in the first 1/5th of lines because every 5th used line endpoint is used the first time around.

The $\alpha = 6$ image shows the first and last lines drawn by setting *Lines Drawn* = 2 and *Step Back* once from 0 to *Drawing progress* = 195. Ends differ by 6 points on the 2-3 point curve. These lines are the last line of the **outer** 0-2 curve (line 196) and **outer** 0-3 (line 1) curves. If you step forward from here, note that line 2 is part of the **inner** 0-3 curve.

The distance between curves. The curves get farther apart from one another as α increases. When $\alpha = 1$ the two curves appear virtually identical but if you view a larger version of this image, they are not. The curves meet at the star's vertices and towards the middle they sag according to how much slack is in each curve.

A chain analogy: Suppose you want to hang a chain across an opening between hooks on posts that are 10 feet apart.

You have a chain that is a bit more than twice that size, but it is connected to itself in a loop and there are an odd number of links in the looped chain, $T = 2k+1$. If you hook a link of the chain to one post and stretch it across the opening you will be able to find a link to hook to the other side where there are k links in one "half" and $k+1$ links in the other. The curves created by both halves will be virtually identical but if you look carefully, the one with k links will be a bit higher (have less curvature) than the one with $k+1$ links. If you go to a post and reduce the number of links on the small side by 1 the two curves will have length $k-1$ and $k+2$. The curves change a bit. The shorter will be a bit less-curved, and the larger will be a bit more-curved, and the distance between them will increase.

The $\alpha = 12$ image with *Drawing progress* = 37 shows lines 38 and 39. This starting point was shown because both lines share the "1" vertex of the 5-point star. As such it provides a chance to see the first (and last) line of a couple of curves. If you change to *Drawn Lines* = 1 you will get less confused by which line is which.

Line 38 (lower line) is the last line in the outer 3-1 curve and the first line in the inner 1-4 curve (*Step Back* to 35 and note that line 36 is part of the outer 3-1 curve; *Step Forward* to 39 and note that line 40 is part of the inner 1-4 curve).

Line 39 (upper line) is the first line of the outer 1-4 curve and the last line of the inner 3-1 curve. If you set *Drawing progress* = 40 you will note line 41 is also part of the outer 1-4 curve but if you set it to *Drawing progress* = 36 you will note that line 37 is part of the inner 3-1 curve.

This image has only a couple of lines on each of the curves because the density is 1/12th each time around for the reasons discussed above.

A final point to note is that the lines alternate between inner curve and outer curve with every other drawn line unless the curve under consideration changes endpoints, just like with the [point and line curves](#) analysis using [two footballs](#).

A limit on the outer curves. There is a limit to reducing the number of links on the short side of a chain loop. The limiting case is a straight line. The same goes for these images.

The 169-line $\alpha = 33$ image is shown with *Drawn Lines* = 1 and *Drawing progress* = 84. The 85th line is at the drawing midpoint (169/2 = 164.5) and this line IS the limiting case of the outer 4-1 curve. This image appears to have 5 (not 10) curves. To visualize this in context, change to *Fixed Count Line Drawing* mode. If you want to see a bit of curvature in the outer curves, try a smaller value of α . This link takes you to a 179-line $\alpha = 23$ image where the outer curves take about 10 lines to create.

Generalizations. Start with this $\alpha = 4$ two footballs image and change S by 2 to get this $\alpha = 2$ puffy pentagram.

