

Two Footballs Challenge Question

The Two Footballs [explainer](#) proposed a general rule for creating *two footballs* images. That rule was:

A general rule. Base *two footballs* images off of J . These images occur when $n = P = 3J \pm 1$ and $S = 2J \pm 2$ but are most visible when J is not too small.

This rule keys off of J . Four images were shown but three images were of roughly the same size in terms of number of lines (determined by S) and number of vertices in the underlying polygon, n . These challenge questions ask you to focus your attention on n rather than J .

The images shown there were created to highlight the different number of cycles that are possible by following the *two footballs* rule.

The values of n shown there were: $n = 247$ ([top right](#)); $n = 248$ ([bottom right](#)); and $n = 250$ ([middle right](#)).

Noticeably absent is $n = 249$. Of course, this may just be a byproduct of choosing images that show the various cycle outcomes shown in the explainer. Or is there something more going on?

Q1) Is it possible to find values of S and J satisfying the *two footballs* rule which has $n = 249$?

Q2) Provide a general condition on n that guarantees that no S and J can be found that satisfy the *two footballs* rule and is consistent with that value of n .