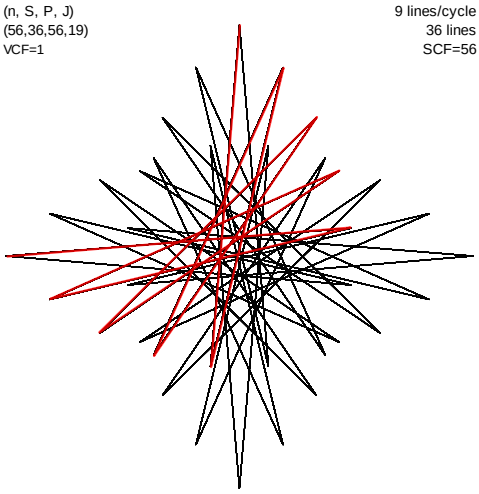

Two Footballs

These images almost demand the use of the web version of the string art file. The largest possible version available using the *Excel* file is to the left which highlights the first cycle. By squinting, one can see the two footballs as well as the inner and outer curves discussed elsewhere, but they are not as readily visible as when ***n***, ***S***, ***P***, and ***J*** are larger. Examples of larger ***n***, ***S***, ***P***, and ***J*** images are discussed below.

How many cycles are there in a *two footballs* image? Recall that a cycle requires returning to one of the vertices of the underlying regular polygon. There MUST BE at least one cycle because the top is always included in the image. There are four pointed ends, two on each football so perhaps all four points are vertices of the underlying regular polygon. If that were the case, there would be 4 cycles (like the top left image). Given that the two footballs are at right angles to one another, and given that both appear to be centered, an image with 4 cycles could only happen if ***n*** is divisible by 4.

If ***n*** is divisible by 2 but not 4, then the vertical football may have both ends pointed (i.e., be the end of a cycle) but the horizontal football's ends are not at vertices so that there are 2 cycles. If ***n*** is odd, then there is only one cycle.

The three versions shown to the right each have roughly the same number of lines.

The top image has ***S*** = 166 lines, with ***n*** = ***P*** = 247 and ***J*** = 82. Because ***S*** is even, one might expect there are 2 cycles of 83 per cycle. But ***n*** is odd so that there is no vertex at the bottom and hence no possibility that the bottom of the vertical football is a vertex of the underlying polygon. Instead, there are two subdivision endpoints that seem to equally create the bottom of that football. This image has a single cycle (GCD(***S***,***P***) = 1).

The middle image has ***S*** = 168 lines, with ***n*** = ***P*** = 250 and ***J*** = 83. Here GCD(***S***,***P***) = 2 so there are two cycles.

The bottom image has ***S*** = 164 lines, with ***n*** = ***P*** = 248 and ***J*** = 83 is the same image used to discuss curves. Here GCD(***S***,***P***) = 4 so there are four cycles.

A general rule. Base *two footballs* images off of ***J***. These images occur when ***n*** = ***P*** = 3***J*** $\pm$ 1 and ***S*** = 2***J*** $\pm$ 2 but are most visible when ***J*** is not too small.