A Quivering Triangle that is actually *Shape-Shifting*

Quivering triangles, *QTs*, have used subdivision endpoints that rotate in three ovals, one at the top, the other two about one third of the way around the circle in each direction. (These ovals are curves created by points, not lines.) The lines that comprise *QTs* stay in one of three tubes. One tube contains lines with endpoints at top and bottom right. A second contains lines from endpoints at bottom left and bottom right. The third contains lines with endpoints at top and bottom left. Other dimension quivering polygons can be described as having this same attribute.

The key to *QTs* is *n* = *P* and *S* = 3*J*. The *QTs* discussed in the quivering polygons explainers are based on *n* = *P* = *S* ± 1 in which case the image is drawn in a single trip around the three point ovals.

Multiple trips around the ovals. If you increase the distance between *n* = *P* and *S*, the number of times around the ovals to complete the image changes. If *n* = *P* = *S* ± *a* then it takes *a* times around as the *a* = 2 upper image *J* = 39, *S* = 117 and *n* = *P* = 115 image shows. You can see the end of the third line is the 2nd point down on the top oval: it takes two times around the oval to complete the image.

If you choose *a* near *J*/2 you can have the first triangle peak near the bottom of the oval like this middle image *QT* which increases *n* = *P* = 137 (*a* = 20), things look different. The image shows *Drawn Lines* = 6. The first triangle's peak is at the bottom of the oval and the second is just to the left of the top.

Even if you use *Drawn Lines* = 3 you will quickly perceive that *there are two triangles quivering here*, each is completing half the oval. One loops counterclockwise down from the top, the other loops counterclockwise up from the bottom.

A *QT* that is also a *3SST*. More interesting is an image I came upon while exploring *n* = *P* models searching for general patterns. The image I found was *n* = *P* = 500, *J* = 185 and *S* = 111. Two images are shown below, *Drawing progress* = 0 at left, and at halfway around the 111 line image at *Drawing progress* = 53 at right (midpoint of 53 and 59 = 56 right at *S*/2).

Although *n* = *P*, *S* is quite clearly not 3*J* so one might imagine that this provides a new set of equations governing *QTs*. The first thing to note is that VCF = 5 so that one can simplify values to (*n, S, P, J*) = (100, 111, 500, 37). Now *S* = 3*J* but *n* is no longer equal to *P*. However, *P* is a multiple of *n* and the image is single cycle. Therefore, it is one of a family of *QTs* with (*n, S, P, J*) = (100, 111, 100*k*, 37) for 1 ≤ *k* < *S*/2 and GCD(*S*, *k*) = 1 as discussed in the single cycle explainer.
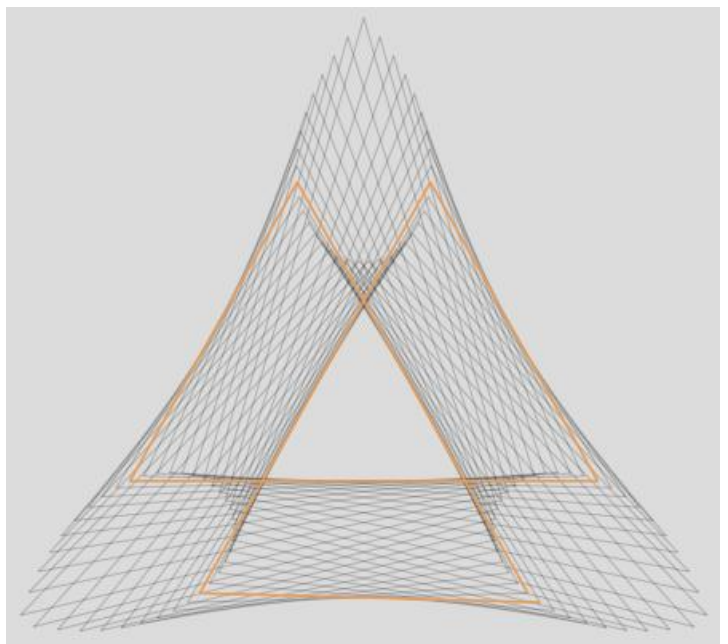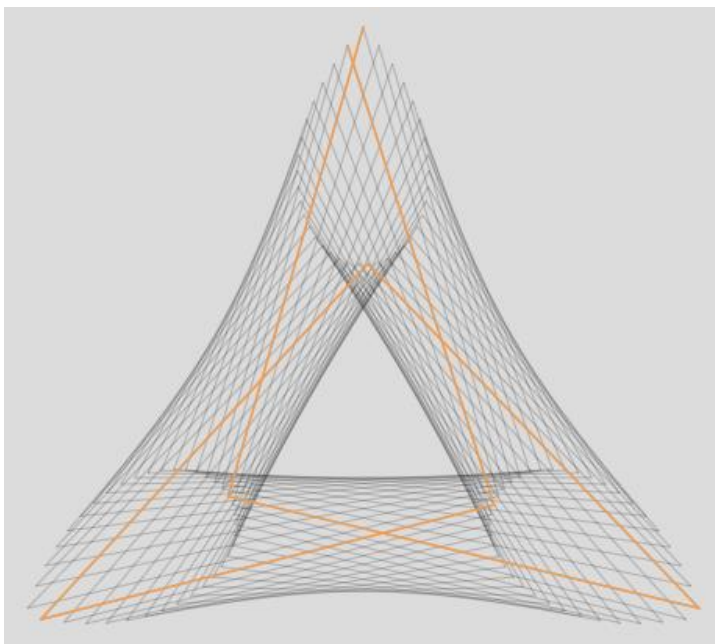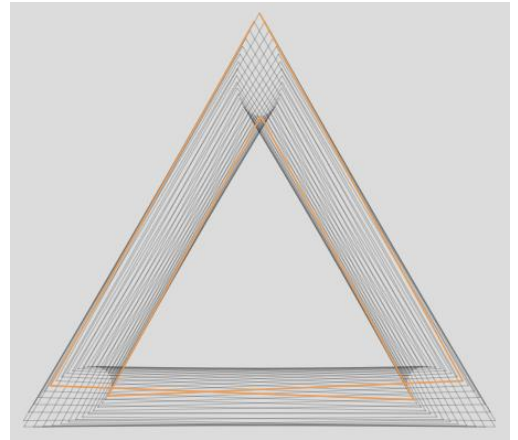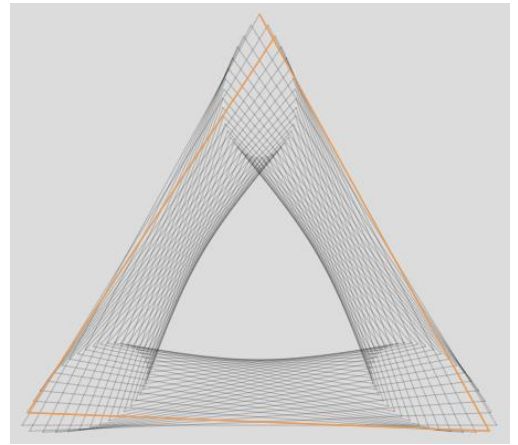
The similarity of the p. 1 bottom _QT_ image with 3SST. If you set _Drawn Lines_ = 6 and use _Single Line Drawing_ (SLD) mode with _Drawing Speed_ = 20 or so, you will see the triangles shift shape from a set of sharply pointed triangles when _Drawing progress_ is small (like at bottom left), to triangles that resemble equilateral triangles when _Drawing progress_ is in the middle (like at bottom right) and back to sharp triangles as _Drawing progress_ approaches **S**.

Differences with 3SST. The 570 line, Three _Shape-Shifting_ Triangles image, is clearly more complex than its _QT_ counterpart. If you follow the above drawing instructions but set _Drawn Lines_ = 7 the triangles that result do change sharpness but there are three substantial differences. **1)** The triangles never look equilateral, there is always a noticeably sharpest angle. **2)** If you follow the peak from a given triangle it does not stay in one area of the image (like the oval at top, lower right, or lower left). Consider for example, the triangle that starts at vertex 0. The peak of that triangle next touches a vertex as the 133rd line drawn (133 = 7·19). This vertex is 13 NOT vertex 0. By contrast, the peaks that start at vertex 0 stay within the oval at the top of the image with _QTs_. **3)** 3SST is single-step, the bottom _QT_ is smallest-step.

Comparing the bottom image to other _QTs_. Quite clearly the triangles change their shape much more radically in the bottom image than if you go through this same process with the middle image. As noted above, this image seems to be drawn by two sets of triangles, one that rotates counterclockwise from the top the other from the bottom, but both maintain their approximately equilateral status. If you set _Drawn Lines_ = 6 with the upper image and watch the triangles quiver you see two approximately equilateral triangles rotating and changing size but dancing quite close to one another – the peaks rotate clockwise around the top oval twice in this instance. Finally, if you examine _Drawn Lines_ = 6 with a 1 × around _QT_ such as taking the upper image and increase **n** = **P** by 1 to **n** = **P** = 116, the only difference is that now the dancing partners are even closer and the image goes around the oval only once (and so gets small only once).

Variations on the theme. As you adjust **k** (i.e., adjust the 100s digit in **P**), interesting patterns emerge. The oval points do not change as **k** changes, the order that those points are used changes as discussed in the single cycle explainer.

   **About the point ovals.** Each is more pointed on one end than the other, so they are not true ovals. Every 3rd line starts in the same oval as 3 lines before so the point ovals can be labelled 0 MOD 3, 1 MOD 3 and 2 MOD 3 based on the location of the start of each line in the image (when **k** = 1). The top oval is 0 MOD 3. The lines that start at 1, 4, 7, … start in the 1 MOD 3 oval (the lower right oval) and those that start at 2, 5, 8, … start in the 2 MOD 3 oval (the lower left oval).

When **k** is divisible by 3 (**k** = 0 MOD 3), all used subdivision points stay within the 0 MOD 3 oval and small images emerge. Other **k** values produce _QTs_ (except **k** = 37 which produces an actual triangle). When **k** = 1 MOD 3, the first line ends in the 1 MOD 3 oval and if **k** = 2 MOD 3 the first line ends in the 2 MOD 3 oval. (So, for example, the first line of the **k** = 5 bottom left image ends in the 2 MOD 3 oval because 5 = 2 MOD 3. Indeed it is about halfway around from its peak.)

   **1×around images.** _One time around does not necessarily mean equilateral._ These sharper than equilateral and equilateral images are both 1 × around because the third line ends a smallest step from the top. Both are _smallest-step_ images with sub-image length 3. Focus on where the other two line segments end. The first segment for the equilateral version ends near the point of the 1 MOD 3 oval, the second is near the point of the 2 MOD 3 oval, and the third is a smallest step from the top. These three lines map out a large "almost equilateral" triangle. By contrast, the sharper angle version's first line ends about 1/3 around ↻ from the point of the 1 MOD 3 oval, the second ends about 2/3 around ↻ from the point of the 2 MOD 3 oval and the third is a smallest step from the top. These three lines map out a "sharply-peaked almost isosceles" triangle. This is about the sharpest triangle you could create since the first and third lines appear to be almost tangent to the inside of their ovals. This version has a squeezed in tube in the middle.

   **Bow-tie QTs.** If tube middles are squeezed as much as possible, the lines will appear to all converge in the center. If you compare the end of the first line drawn in **P** = 800 and **P** = 3800 you will see that they are one point different from one another, 70% ↻ from the point of the 2 MOD 3 oval (**k** = 8 and 38 are both 2 MOD 3), When _Drawing progress_ = 78 for **P** = 800 and _Drawing progress_ =78 for **P** = 3800 in SLOD mode with _Drawn Lines_ = 1 you will see the positively sloped line with steepest slope. Both appear tangent to the left side of the 0 MOD 3 oval and the right side of the 2 MOD 3 oval.

   **Fractionally filling in the ovals.** If the third line ends about 1/2, 1/3 or 1/4 of the way around the 0 MOD 3 oval, then the image will appear as multiple triangles much like the middle image on p. 1. These images will emerge in _Fixed Count Line Drawing_ mode regardless of the value you choose for _Drawn Lines_ but the images are best seen using the setting of _Drawn Lines_ noted: ½, _Drawn Lines = 6_; ⅓, _Drawn Lines = 9, Sharp_; ⅓, _Drawn Lines = 9, Equilateral_; ¼, _Drawn Lines = 12_.