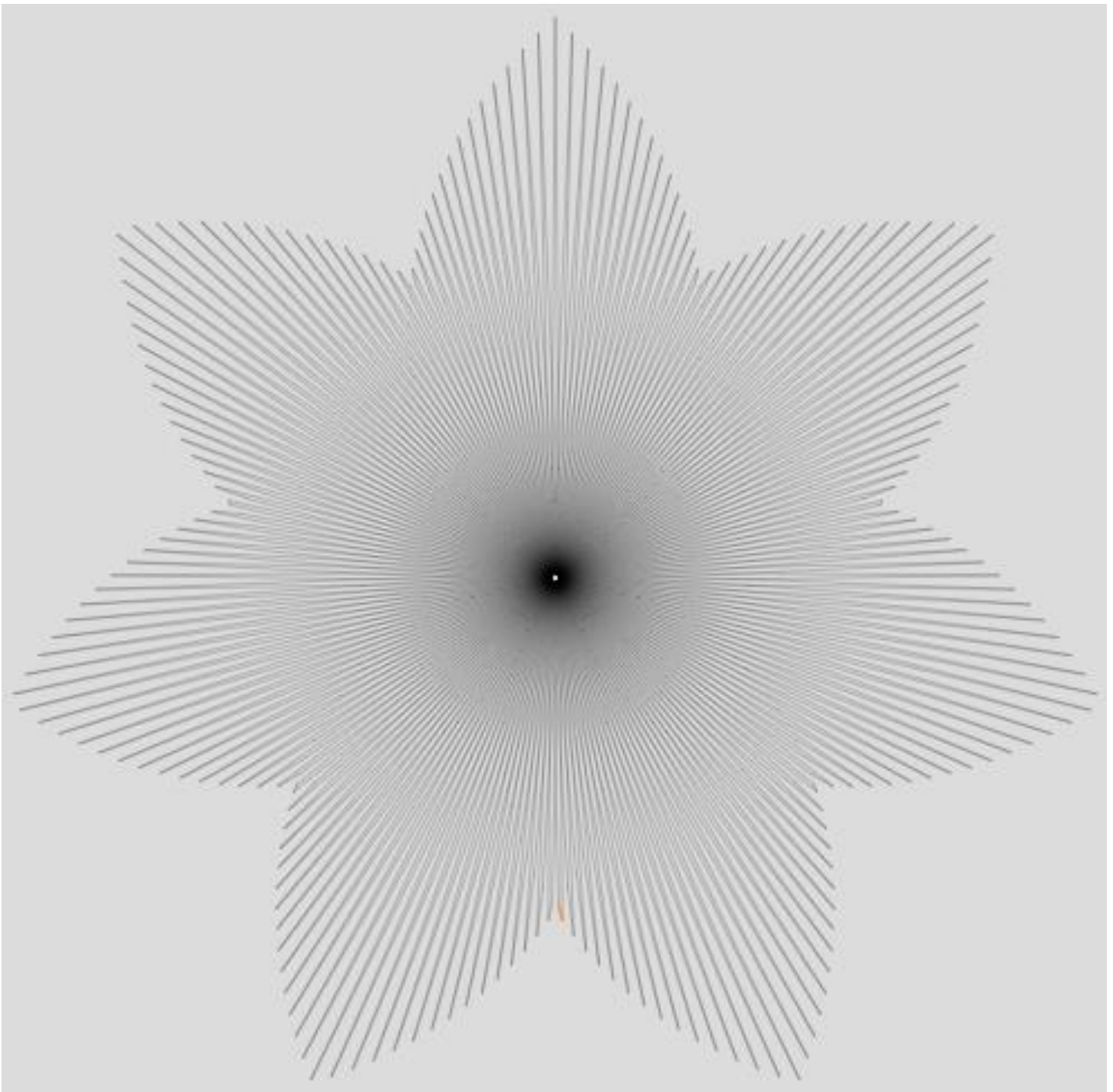
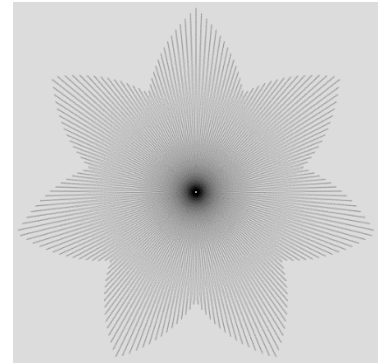


$n = P$ Porcupines

Porcupine polygons have a spiky look, and these two images (from the [Generalized Stars](#) explainer) quite clearly qualify as porcupine. ([The left](#) has 199 spikes; [the right](#), which we focus on here, has 201.)

Traditional porcupine images are almost half-way around meaning P is the closest number to $nS/2$ so that the image is [single-step](#) after 2 lines. Porcupines require $SCF = 1$ so that all subdivisions are used. By

contrast, $n = P$ images use very few of all possible subdivision points and at most S segments. The upper left image is [smallest-step](#) after 2 lines and the upper right (blown up below with one minor addition) is smallest-step after 5 lines.



Overview. This image has $n = P = 201$, $J = 100$ and $S = 499$. Lines that look like they are going from one end to the other just missing the center are NOT single segment lines (like the upper left image). Because $S/3 < P < S/2$, each “line” contains 2 to 3 segments and there is at least one segment on each part of the VF (although often there are two). Even though there are 499 lines used to create this image, it looks like 201 lines. Roughly speaking, smallest-step occurs every 5 segments and each of these smallest-step moves creates two lines. It is worth initially focusing the VF.

This image closely mirrors the VF. The VF is a single-step counterclockwise drawn 201 point star. The first two segments tell the story: the first line of the VF is from 0-100 just to the right of the center and the second is from 100-200 just to the left of the center. Note: 200 is 1 less than n . Subsequent odd segments decline from 100 by 1 and even segments decline from 200 by 1 so the VF is created in $\frac{1}{2}$ \cup rotation as you can see using [this link](#) in *Fixed Count Drawing* mode.

Most of the time, it takes 5 segments to draw two lines. Each segment spans $P = 201$ subdivisions of the VF so that the first two segments end at 402. Since there are 499 segments in a line of the VF, these two segments are on the first line of the VF (or put another way, they are colinear). This looks like a line spanning the image from the top to just right of the valley of the 3-4 scallop. This valley segment is at Level 97 = 499 – 402. The next segment of the image starts at level 97 on the first line of the VF but ends at Level 104 = 201 – 97 of the 2nd VF line from vertex 100 to 200. THIS IS THE SEGMENT highlighted in the large image (which is why the image is large). Two more segments, one ending at 305 on the 2nd VF line and the other ends at 7 of the 3rd VF line. These three segments are NOT colinear but are close enough that they appear so. The five segments create 2 “lines.” These five segments also create a smallest-step.

The point curves are created by Level changes (of 7) per smallest-step. Subsequent 5 segment additions increase Level by 7 each time. The first 14 of these cycles ($70 = 14 \cdot 5$) create the curve from the peak at the top vertex 0 to the valley between 6 and 0 (the 6 here is the last peak of the 7 point star, not the 6th of 201 vertices of the underlying polygon). The 70th line is at Level 98 ($98 = 14 \cdot 7$). Similarly, the first two segments of each 5 segment set start at Level 97 (as noted above) then decreases by 7 each 5 segments from here (so note line 7 ends at Level 90 and 12 ends at Level 83, ..., and line 67 ends at Level 6). Peak 3 occurs with line 72 ending at Level 1 (since a decrease of 7 levels from Level 6 is Level 1).

A modified version of the *Tracking lines Excel* file from 2.3 is provided which shows all 499 line segments. The first 80 are shown below, highlighted to note changes in direction (which are smallest Levels). The table to right summarizes this information. Note that the Level information in the previous two paragraphs is summarized in the Point of star 0 and 3 columns. It is worth noting that, as expected, this summary is symmetric about the vertical line.

Summary of Tracking Lines Locations Excel file	Point of star	3	6	2	5	1	4	0
Peak line	72	144	216	283	355	427	499	
Level	1	2	3	3	2	1	0	
Level just before peak (peak-5):	6	5	4	10	9	8	7	
Level just after peak (peak+5)	8	9	10	4	5	6	7	
Opposite valley line before peak	70	142	214	281	353	425	497	
Level there	98	99	100	94	95	96	97	
Opposite valley line after peak	74	146	218	285	357	429	2	
Level there	96	95	94	100	99	98	97	

NOTE: If using *SLOD* mode with *Drawn Lines* = 1, set *Drawing progress* at 1 less than line number to see the line (*Drawing progress* is start of the line). Lines to next peak (sum to 499) 72 72 72 67 72 72 72

Near peaks, it only takes 4 segments to draw two lines. The clearest way to show this is to set *Drawn Lines* = 4 and *Step Back* two steps so that the last two and first two segments are shown using *Single Lines Overlaid Drawing* mode (with *Drawing progress* = 497). This shows the first and last line of the image since the last two are symmetric to the first two and are thus both collinear. One finds additional 2 lines in 4 segments by setting *Drawing progress* = Peak line-2 using the table above. (In the *Excel* file, these boxed points are where the highlighting changes to the new pattern that holds until the next peak occurs. For example, multiples of 5 are highlighted green to 70 but 74 starts a new pattern in yellow.)

Small segments. Near the peaks the side with 3 segments has small level changes. The smallest change near each peak is shown to right. Note that these level changes also are symmetric. If you think the 7 change in Level line is small, check out the size of the line starting at *Drawing progress* = 213 or 285.

Smallest level change segments in the valley of each petal opposite star point listed														
Star point	0	3	6	2	5	1	4							
Line k	2	3	69	70	141	142	213	214	285	286	357	358	429	430
Sub End	402	104	396	98	397	99	398	100	399	101	400	102	401	103
Segment	0.806	1.208	27.8	28.2	56.8	57.2	85.8	86.2	114.8	115.2	143.8	144.2	172.8	173.2
VF Start	0	100	87	187	173	72	58	158	144	43	29	129	115	14
VF Stop	100	200	187	86	72	172	158	57	43	143	129	28	14	114
Level	97	104	103	98	102	99	101	100	101	99	102	101	98	103
Δ Level	104	7	92	5	94	3	96	1	98	1	100	3	102	5

A bit more on Levels. Recall that Level 0 points are vertices of the underlying polygon with larger Level numbers being closer to the center of the circle. Levels are created from the concentric circles of subdivision points and these circles are not equally spaced as can readily be seen in the [Levels](#) explainer. This is why the point curves based on points that are 7 Level changes from one point to another ($7 = 5 \cdot P - 2 \cdot S = 1005 - 998$) appear as curves rather than straight lines.

Peaks and Valleys. Looking carefully, you will note that the 7 peaks differ regarding how pointy they are. This has to do with the image being single cycle. The top (0) is clearly the sharpest. Opposite peak 0, the valley is the shallowest (Level 97). Peaks 3 and 4 are at Level 1 and the largest valley number is at Level 98. Peaks 1 and 6 are at Level 2 with largest valley number at Level 99. Finally, Peaks 2 and 5 have peaks at Level 3 (and an adjacent ending point at Level 4) which is why these two peaks appear most rounded. By contrast, the valleys opposite these peaks are deepest at Level 100 (Level 100 and 101 are paired at lines 213, 214 and again at 285, 286). These findings are also summarized in the tables on p. 2.

MA. Tracking lines. This analysis has leaned heavily on the *Tracking Lines in the First Cycle* Excel file from section 2.3. This file was developed to analyze [spinning needle stars](#). The new version, *Tracking Line Locations*, has been modified to include a much larger cycle (of 560 lines) and is separately posted in the $n = P$ section 2.10. The first 80 lines are shown so that one can see what happens near the first drawn peak (3) at line 72, boxed below.

201	n	The Vertex Frame (VF) start and stop values are the RED																		Sub End = $\text{MOD}(k \cdot P, S)$	
499	S	numbered vertices of the polygon.																		Segment = $k \cdot P / S$	
201	P	Polygon vertices are in RED																		VF Start = $\text{MOD}(\text{INTEGER}(\text{Segment}) \cdot J, n)$	
100	J	Endpoints of first cycle segments are labelled by color:																		VF Stop = $\text{MOD}(\text{VF Start} + J, n)$	
		Non 5th in Blue									5th endpoints in GREEN									Level = $\text{MIN}(\text{Sub End}, S - \text{Sub End})$.	
Line k		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Sub End		201	402	104	305	7	208	409	111	312	14	215	416	118	319	21	222	423	125	326	28
Segment		0.40	0.806	1.208	1.611	2.014	2.417	2.82	3.222	3.625	4.028	4.431	4.834	5.236	5.639	6.042	6.4	6.8	7.3	7.7	8.1
VF Start		0	0	100	100	200	200	200	99	99	199	199	199	98	98	198	198	198	97	97	197
VF Stop		100	100	200	200	99	99	99	199	199	98	98	98	198	198	97	97	97	197	197	96
Level		201	97	104	194	7	208	90	111	187	14	215	83	118	180	21	222	76	125	173	28
Δ Level		201	104	7	90	187	201	118	21	76	173	201	132	35	62	159	201	146	49	48	145
				small																	
Line k		21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Sub End		229	430	132	333	35	236	437	139	340	42	243	444	146	347	49	250	451	153	354	56
Segment		8.5	8.9	9.3	9.7	10.1	10.5	10.9	11.3	11.7	12.1	12.5	12.9	13.3	13.7	14.1	14.5	14.9	15.3	15.7	16.1
VF Start		197	197	96	96	196	196	196	95	95	195	195	195	94	94	194	194	194	93	93	193
VF Stop		96	96	196	196	95	95	95	195	195	94	94	94	194	194	93	93	93	193	193	92
Level		229	69	132	166	35	236	62	139	159	42	243	55	146	152	49	249	48	153	145	56
Δ Level		201	160	63	34	131	201	174	77	20	117	201	188	91	6	103	200	201	105	8	89
															small change but straddles middle						
Line k		41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Sub End		257	458	160	361	63	264	465	167	368	70	271	472	174	375	77	278	479	181	382	84
Segment		16.5	16.9	17.3	17.7	18.1	18.5	18.9	19.3	19.7	20.1	20.5	20.9	21.3	21.8	22.2	22.6	23.0	23.4	23.8	24.2
VF Start		193	193	92	92	192	192	192	91	91	191	191	191	90	90	190	190	190	89	89	189
VF Stop		92	92	192	192	91	91	91	191	191	90	90	90	190	190	89	89	89	189	189	88
Level		242	41	160	138	63	235	34	167	131	70	228	27	174	124	77	221	20	181	117	84
Δ Level		186	201	119	22	75	172	201	133	36	61	158	201	147	50	47	144	201	161	64	33
Line k		61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Sub End		285	486	188	389	91	292	493	195	396	98	299	1	202	403	105	306	8	209	410	112
Segment		24.6	25.0	25.4	25.8	26.2	26.6	27.0	27.4	27.8	28.2	28.6	29.0	29.4	29.8	30.2	30.6	31.0	31.4	31.8	32.2
VF Start		189	189	88	88	188	188	188	87	87	187	187	86	86	86	186	186	85	85	85	185
VF Stop		88	88	188	188	87	87	87	187	187	86	86	186	186	186	85	85	185	185	185	84
Level		214	13	188	110	91	207	6	195	103	98	200	1	202	96	105	193	8	209	89	112
Δ Level		130	201	175	78	19	116	201	189	92	5	102	199	201	106	9	88	185	201	120	23
										small			PEAK 3			small					