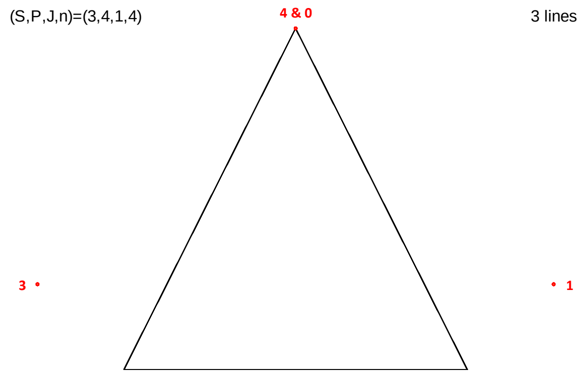
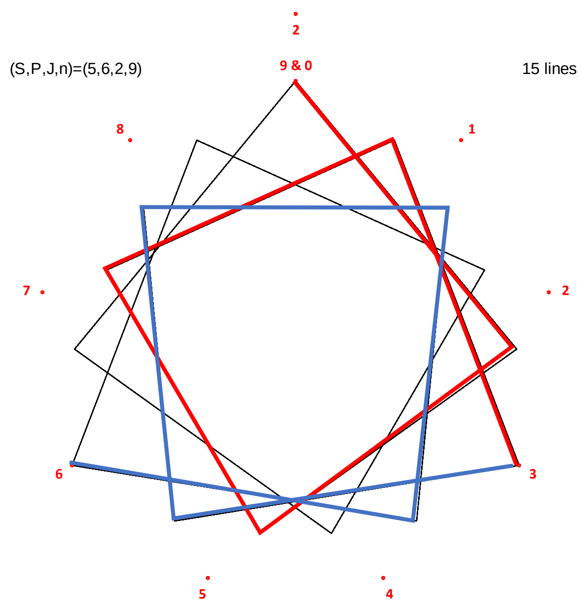
The Number of Segments in a Cycle and the Number of Cycles in a Circuit

A completed image includes at least one vertex from the parent polygon because a circuit is completed once the top of the polygon (the starting point of the image) is used as a segment end point. As can be seen in the (***S***, ***P***, ***J***, ***n***) = (3, 4, 1, 4) top image, the top vertex may be the only vertex of the parent square included in the final triangular image. It is important here to not confuse the triangle's vertices with the parent polygon's vertices.



If more than one of the parent polygonal vertices are included in the final image, then the image is made up of multiple cycles. *A cycle,* ***C****, is the smallest number of connected segments that are required to achieve a vertex of the polygon as an endpoint of a segment.* An image is created from one or more cycles.

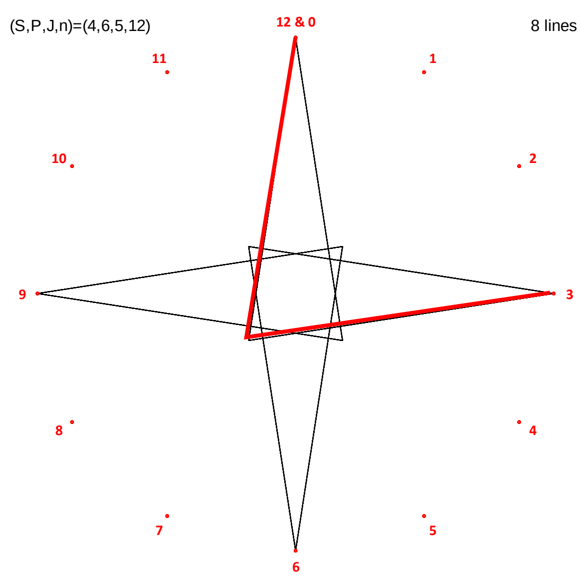
The cycle in the (***S***, ***P***, ***J***, ***n***) = (5, 6, 2, 9) middle image is ***C*** = 5. The image is made up of 3 such cycles: the first is shown in **red**, the second is shown in **blue**, the third is in black (starting at vertex 6 and ending at 9&0). (By contrast, the top image has one cycle of ***C*** = 3.)



Each cycle is a rotated version of other cycles. The **blue cycle** is simply the **red cycle** rotated clockwise by 120° (or 3/9 = 1/3 of a full rotation). The black cycle is a 2/3 clockwise rotation of **red** (or a 1/3 counterclockwise rotation). Put another way, *the lines in a single cycle shows us the essential character of the image.*

It is worth looking at the locations on the vertex frame of **red's** cycle end points. Remember, here, ***S*** = 5 and ***P*** = 6. The first point is **1/5** of the way along the second part of the vertex frame (from 2 to 4). The second point is **2/5** of the way along the third line of the vertex frame (from 4 to 6). The third point is **3/5** of the way along the fourth line of the vertex frame (from 6 to 8). The fourth point is **4/5** of the way along the fifth line of the vertex frame (from 8 to 1). The fifth point is **5/5** of the way along the sixth line of the vertex frame (from 1 to 3) or point 3 of the 9-gon. Each subdivision fraction, **1/5** to **5/5**, is used.

Note that ***C*** = ***S*** in the two above examples. This need not be the case as we can see in the (***S***, ***P***, ***J***, ***n***) = (4, 6, 5, 12) bottom image. The first cycle is shown in **red**. The first end point is 2/4 of the way on the second vertex frame line (from 5 to 10) and the second end point is 4/4 of the way on the third vertex frame line (from 10 to 3) at point 3 so that ***C*** = 2. The *number of segments in a cycle*, ***C***, is given by:



$$C = \frac{S}{\mathrm{GCD}(S,P)}$$ where **GCD** is the greatest common divisor function.

The *number of cycles in the image*, ***M***, is given by:

$$M = \frac{L}{C}$$ where ***L*** is the number of lines in the image.

(As discussed in 2.2a, ***L*** = (***S********n***/**VCF**)/**SCF** with **VCF** = **GCD**(***n***, ***J***) and **SCF** = **GCD**(***S********n***/**VCF**, ***P***).)

The number of cycles is the same as the number of vertices of the parent polygon used in the image.