A Primer on Vertical Symmetry

String Art images always exhibit vertical symmetry because the vertex frame (VF) is a regular star or polygon and, as such, has vertical symmetry. [This image was chosen with large enough *n*, *S*, *P*, and *J* that you should not worry about counting the numbers involved. A link below reveals these values.]

There are a total of $\mathbf{v}_{used} = \mathbf{n}/\text{VCF}$ lines in the VF. Call these lines $\mathbf{1}, \mathbf{2}, \dots, \mathbf{v}_{used}$ Each line has \mathbf{S} subdivisions so there are $\mathbf{T} = \mathbf{S} \cdot \mathbf{v}_{used}$ subdivisions. Consider the first and last line of the VF.

1. The first line of the VF goes from vertex **n&0** to vertex **J**.

 v_{used} . The last line of the VF goes from vertex n-J to vertex n & 0.

Reason. Because the VF is a complete continuously-drawn star, the end of the last line in the VF is the vertex **n&0**. Each line in the VF spans **J** vertices so the start of that line must have started at vertex **n-J** as shown in Fig. 1.

These lines are mirror images of one another through the dashed blue vertical line from top vertex through the center of the polygon. That is, these lines exhibit *vertical symmetry*.

Both lines have **S** subdivisions and the start of the first is end of the last as both are at the top vertex as shown in Fig. 2. Because each VF line spans the same number of vertices, both are the same size and therefore subdivisions are the same size on each line of the VF.

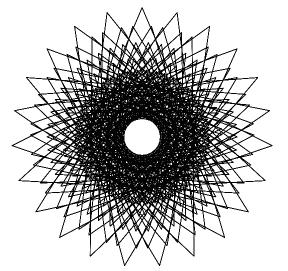
The end of the first segment (at 1) and the start of the T^{th} segment (at T-1) are at the same location on their respective lines.

This same sameness occurs for other subdivision segments on the first VF line with the end of the S^{th} being at vertex J, just as the start of the $T-S^{th}$ segment at vertex n-J in Figures 1 and 2.

Put another way, these subdivisions exhibit vertical symmetry as well.

The same positioning is true for other VF segments, and for the subdivisions on each segment on these lines on the second through \mathbf{v}_{used}^{th} line. All VF lines and subdivision segments exhibit vertical symmetry.

Given this symmetry, the P^{th} subdivision endpoint and the start of the $T-P^{th}$ starting point are symmetric across the vertical line as shown in Fig. 3. For ease of exposition, this shows P = S+1.



The image to the left is the result of continuing this process of drawing lines every P^{th} subdivision. This image would be the same whether the first line is at P or T-P.

To see this <u>253-line image</u> get drawn, use the *Fixed Count Line Drawing* option in *Drawing Mode* and adjust the *Drawing Speed* to suit your preferences.

