About Implied Exterior Angles

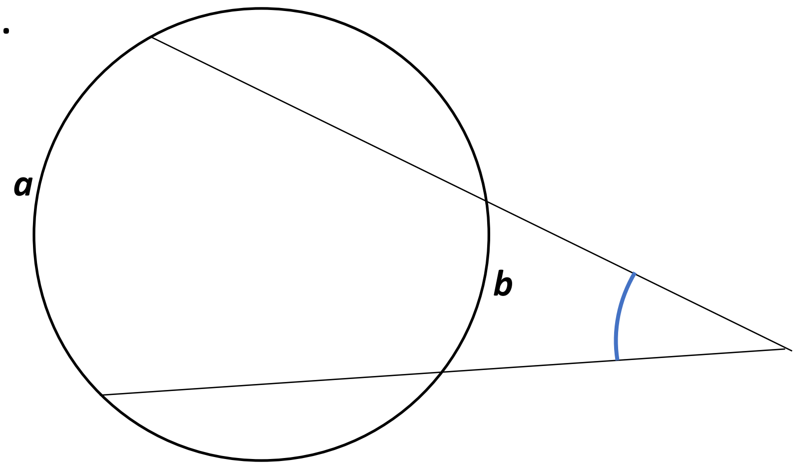
If two lines spanning vertices a regular n-gon do not intersect there are one or more vertices between the two lines on each side. Let those number of vertices be a and b. One of two things must be the case.

A. The lines are parallel.

B. The lines are not parallel.

If the lines are parallel, the lines are an equal number of vertices apart or $a = b$. Otherwise, one is larger. Let that be a so that $a > b$. This is the situation shown as **I**. below. If the lines between vertices are extended, they will eventually intersect on the smaller side. The angle thus created is called an *Implied Exterior Angle* since it is exterior to the circle.

I.



The size of the implied exterior angle is equal to the difference between a and b times $180/n$.

Alternatively, if a and b represent arcs of the circle and central angles, then the implied exterior angle is half the difference between a and b.

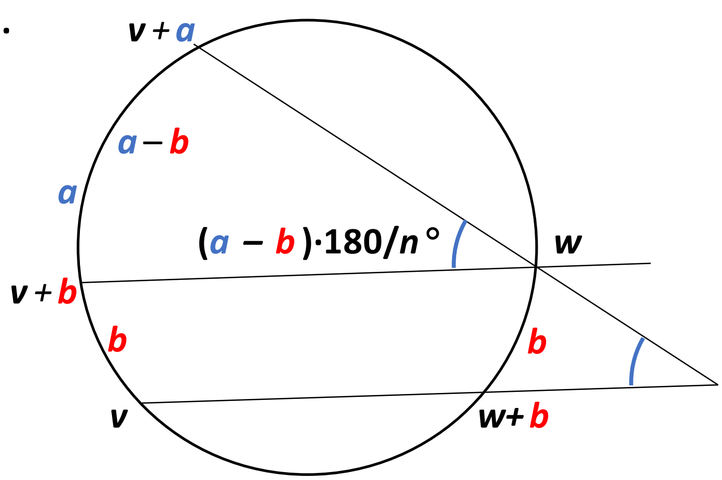
A numerical example. Let $a = 11$, $b = 5$, and $n = 36$. The implied exterior angle is $(11\text{-}5)\cdot 180/36 = 30°$.

Alternatively, If $a = 110°$, $b = 50°$ then the implied exterior angle is $(110\text{-}50)/2 = 30°$.

These answers are the same because a 1 vertex arc of a 36-gon is 10°, so an 11 vertex arc is 110°.

Why this formula works. In Figure **II**, let v and w be vertices of an n-gon and a and b are numbers greater than zero where $a+b < n\text{-}1$ (so that each line spans at least one vertex) and $a > b$. Since $a > b$, we can add b to v and the line between $v+b$ and w is parallel to the line between v and $w+b$ since each of the two lines spans b vertices and are hence parallel.

II.



This newly constructed line has the same angle as the original angle, but this angle is an inscribed angle. This angle has size $(a - b)\cdot 180/n°$, the same as the implied exterior angle because complementary angles are the same.

If instead, a and b are arcs or central angles, then the implied exterior angle is $(a\text{-}b)/2$.