

Inscribed Angles and Central Angles

Arcs of a Circle and Central Angles. Connecting any three vertices of a regular n -gon produces a triangular image. One can easily describe the size of the angles of this triangle by counting the number of vertices between points, a , b , and c . By construction, $a + b + c = n$ in the bottom left image.

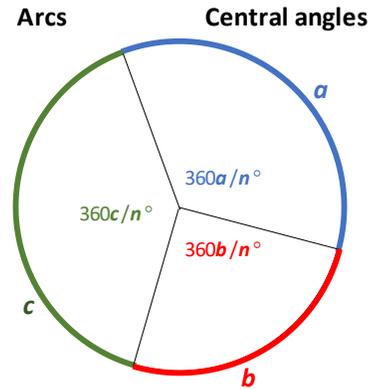
One can think of a , b , and c as *arcs of a circle* that cumulatively create the circle. One can also conceptualize these as *central angles of a circle* created by connecting endpoints of each arc to the center of the circle (via three radii as shown to the right).

Each arc has central angle of: $360a/n^\circ$, $360b/n^\circ$, and $360c/n^\circ$.

The sum of these central angles is 360° .

The triangle angles are: $180a/n^\circ$, $180b/n^\circ$, and $180c/n^\circ$.

Note that each of these angles is half the size of its associated central angle and that, as expected, the sum of angles is 180° . Each of these angles is an example of an *inscribed angle*.



Two intersecting line segments (chords of a circle) create angles in one of two ways: *Inscribed Angles* and *Interior Angles*. Inscribed angles occur if the intersection is on the circle while interior angles occur if the intersection is on the interior of the segments. Interior angles are discussed elsewhere.

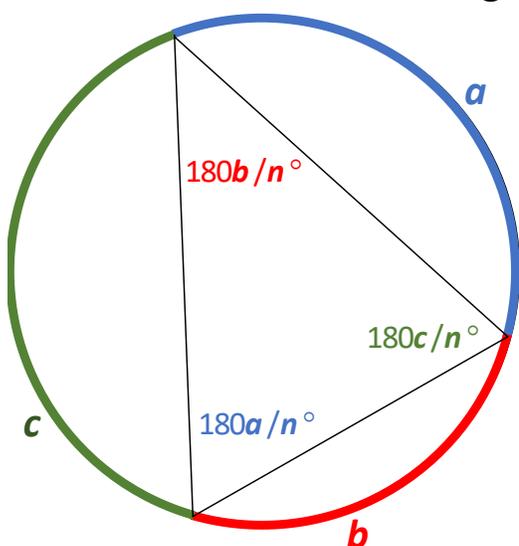
Inscribed Angles. If an angle is formed from 3 points on a circle, the angle formed is called an *inscribed angle*. If the three points are vertices of a regular n -gon then the angle created is determined by the number of vertices between points relative to n . In the bottom right image, let $a = |v_a - v_b|$ where v_a and v_b are vertices of the regular n -gon.

The angle created using these vertices as ends of the legs will be the same regardless of where the third vertex is on the circle (as long as it is not between v_a and v_b).

The image shows three such examples at points t , u , and w on the perimeter of the circle. The angle in each case is half the size of the central angle according to the *Inscribed Angle Theorem*. The central angle is $360a/n^\circ$. The inscribed angle is $180a/n^\circ$.

Counting Vertices. When creating angles from polygonal vertices the number of polygonal vertices between the used vertices determines the angle in question. After that, multiply by $360/n$ or $180/n$ to get the central or inscribed angle measured in degrees. Therefore, it is often easiest to focus on counting the vertices between *used* vertices instead of thinking of angles in degree terms.

Arcs **Inscribed angles**



Inscribed angles

