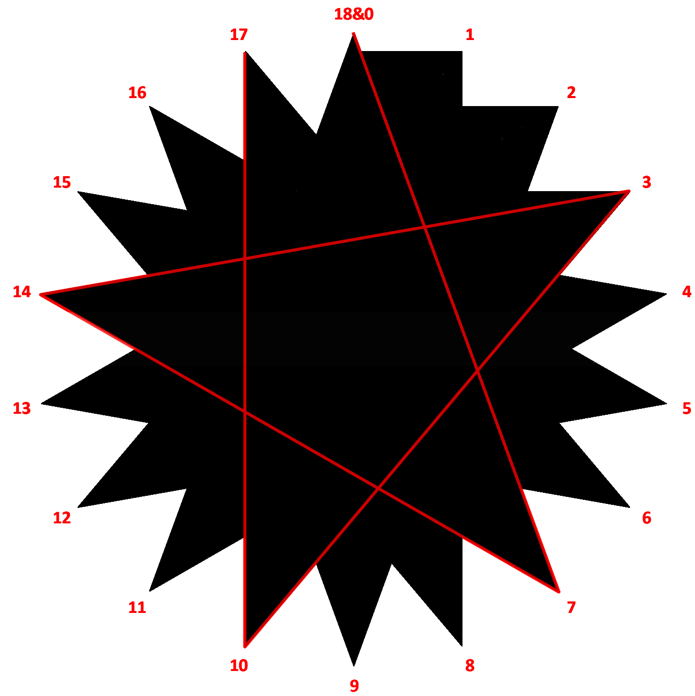
What is Regular about the Angles in Rotating Polygons and Stars?



These images are examples of [Rotating Polygons and Stars]().

***Top R**ight.* The first 5 lines of an 18,7-star are shown in **red**. These lines show a 5,2-star cracked open between vertex **17** and **0** appears counterclockwise (↺)-drawn.

***Bottom L**eft.* The first 7 lines of an 18,5-star are shown in **red**. These lines show a 7,2-star also cracked open between vertex **17** and **0** that will appear ↺-drawn.

***Bottom R**ight.* The first 3 lines of a 20,7-star are shown in **red**. These lines show a triangle "over-closed" between vertex **1** and **0** that will appear clockwise (↻)-drawn.

Claim. Each of the angles in these "almost" polygons or stars that rotate to create an ***n*,*J***-gram are equal EXCEPT for the point that is **<u>not</u>** part of the polygon.

The fact that all but one angle is the same follows from the *Inscribed Angle Theorem*. In our context, the size of the angle is 180/***n*** times the number of open side vertices spanned (***n*** – 2***J***). These images were chosen because all angles are multiples of 10° for ***n*** = 18 and multiples of 9° for ***n*** = 20.

TR. As discussed elsewhere, a regular 5,2-star has 36° angles as opposed to 40° at vertices **4**, **7**, **10**, and **13**. There is formally no angle for the fifth point but if you extend the **0-7 and 10-17** lines they intersect outside (and above) the polygon because 3, the span between **7** and **10**, is larger than 1, the span between **17** and **0**. The angle resulting from this intersection, the *Implied Exterior Angle*, is 20° = 180·(3-1)/18. (Note that 5·36 = 180 = 4·40 + 20.)

BL. A regular 7,2-star has 180·3/7 = 77.14° angles as opposed to 80° at vertices **2**, **5**, **7**, **10**, **12**, and **15**. The *Implied Exterior Angle*, is 180·(7-1)/18 = 10·6 = 60°. (Note that 7·180·3/7 = 540 = 6·80 + 60.)

BR. An equilateral triangle has 60° angles as opposed to 54° = 9°·6 at vertices **7**, and **14**. The interior angle between **0** and **1** is 180°·(7+1)/20 = 9°·8 = 72° according to the *Interior Angle Theorem*. (Note that 3·60 = 180 = 2·54 + 72.)

A general rule. The vertex angles of a rotating image are larger than its regular counterpart for ↺-drawn images (and its implied exterior angle is smaller), but smaller for ↻-drawn images (with a larger interior angle).