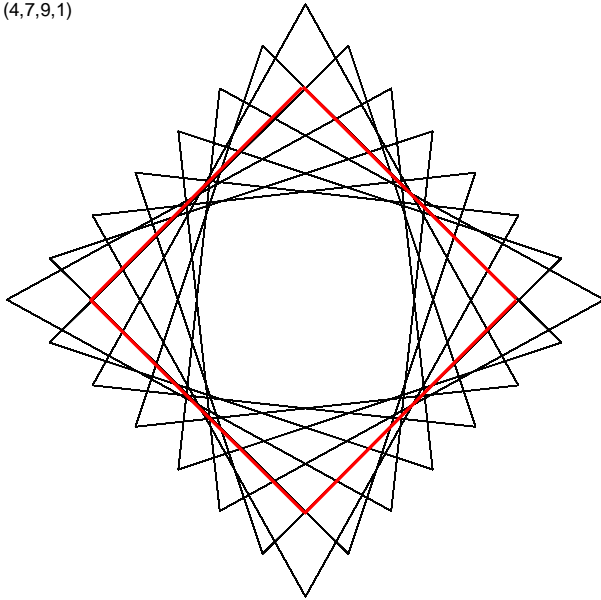


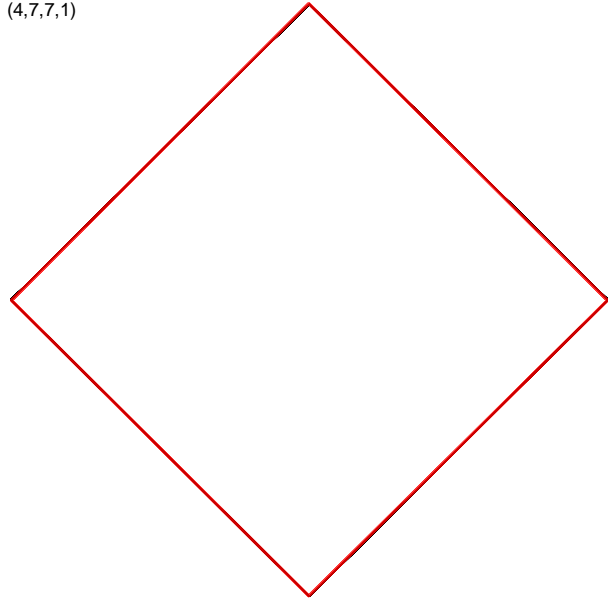
## Calculating Areas of Squares in Modified Brunes Stars

Modified Brunes Stars occur if  $n = 4$ ,  $J = 1$  and  $2 < S < P < 2S$  with  $SCF = 1$ . The left image is an example.

(4,7,9,1)



(4,7,7,1)



The four corners of each image at vertices 0, 1, 2, and 3 are (0, 1), (1, 0), (0, -1) and (-1, 0).

1. Compare the areas of the two squares highlighted above.

- Is the square on the left more or less than half the size of the one on the right?
- Provide exact area calculations of each square in square units based on the coordinates above.  
(Hint: This can be done without resorting to square roots.)

2. Consider the first line in the figure at left above, from vertex 0 to the second subdivision on the second segment of the vertex frame (since  $P = 9 = 7+2 = S+2$ ). There are two lines perpendicular to this line, one of which starts at the third vertex and ends at subdivision 2 on the first line of the vertex frame. This intersection appears below and to the right of the top red right angle in the left image.

- Find equations for the two lines [it may help to use point-slope form of the line  $y-y_0 = m(x-x_0)$ ].
- Find the intersection point in  $(x, y)$  form using these equations.
- Find the distance from this point to the center of the image using the *Pythagorean Theorem* [which is easy to do in this case since one of the points is (0, 0):  $d = (x^2 + y^2)^{0.5}$ ].
- Is the square you just found one vertex of larger or smaller than the one in red?

3. The largest internal square from Modified Brunes stars where  $S$  is odd occurs when  $P = S+2$  and the vertices of the internal square are on the axes (the image at left is an example given  $S = 7$ ).

- What is the smallest value of  $S$  that produces an internal square that is at least  $\frac{3}{4}$  of the size of the square on the right above?
- What is the smallest value of  $S$  that produces an internal square that is at least 90% of the size of the square on the right above?

**HINT:** This is easiest to do by modelling your answer in Excel. Think through the steps on paper, but then let Excel do the work for you.