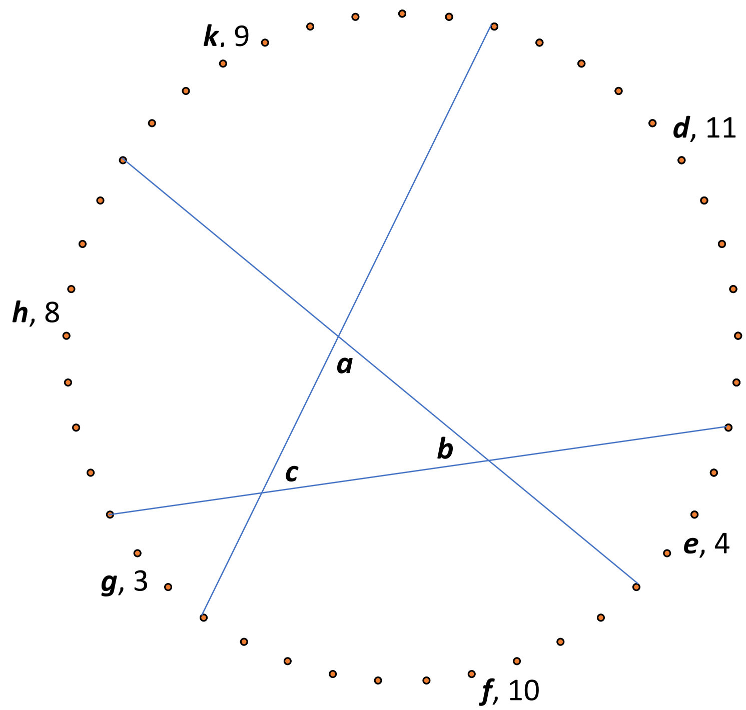
Calculating Δ Angles using Vertices



Three non-parallel lines will eventually intersect one another. Unless they intersect at a single point, they inevitably create a triangle.

If the lines are from vertex to vertex of an n-gon and if each line intersects the other two lines at two distinct points on the interior of each segment, then an image like the one at the right occurs. The interior triangle has three angles, a, b, and c.

The three lines create six non-overlapping arcs of the circle. Each arc spans one or more vertices. The arcs are labelled from d to h and k, and the number of vertices spanned by each arc is also noted.

This image has $n = 45$, the sum of the six numbers. This was chosen so that each angle is a multiple of 4 (because $4 = 180/n = 180/45$). Angles are obtained using the *Interior Angle Theorem* which is $180/n$ times the sum of opposing arcs, or: $a = 4\cdot(k+f) = 4\cdot(9+10) = 76°$; $b = 4\cdot(e+h) = 4\cdot(4+8) = 48°$; $c = 4\cdot(d+g) = 4\cdot(11+3) = 56°$.

The lines above were chosen so that different values occurred for each arc. Single-jump stars would not show this variety as the sum of three consecutive values would have to be J or n-J (as with the **red** and **blue** sets of lines below).

Finding triangles inside stars. The *Analyzing Stars inside a Star* explainer showed that an n,J-star had J-2 internal n-stars and an internal n-gon at the center of the image. Of course, once n and J become somewhat "large" the internal stars are difficult to pick out of the overall image. One might ask: *Where and how many **equilateral triangles, ▲**s, are below?*

Consider the bottom left 60,29-star. Embedded in that central mass are ▲s, based on a 60,20-star. Two of the 20 ▲s that comprise this 60,20-star are shown to the right. These are from three sets of parallel lines drawn at 10-line intervals starting with **blue line 1** from vertex **0** to **29** and noted as: **1 0-29**; **11 50-19**; **21 40-9**; **31 30-59**; **41 20-49**; **51 10-39**. Each internal 6,2-star creates 8 ▲s total, the two large ones and six surrounding smaller ones. In all the left image has 80 ▲s.