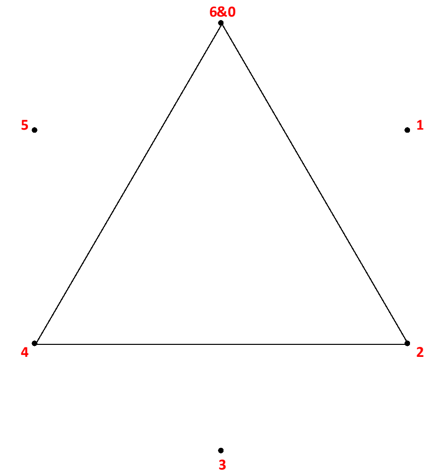
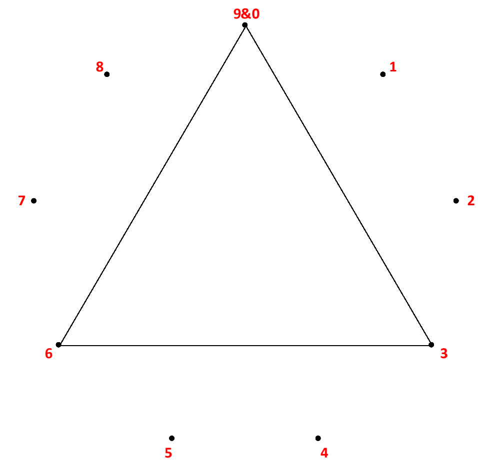
How many points does a continuously-drawn *n*,*J*-star have?



On Commonality. The reason there is no continuously-drawn 6-point star is that 2 is the only number between 1 and 3 = 6/2 and 2 has a factor in common with 6 (namely 2). The 6,2-star is an equilateral triangle connecting even vertices **6&0**-**2**-**4**-**6&0** as seen to the right. The three odd vertices (**1**, **3**, and **5**) are simply not part of the "star" that has been created.



The same argument could be made for a 9,3-star except that now the only used vertices are **9&0**-**3**-**6**-**9&0** or multiples of 3 because 9 and 3 have a common factor of 3. In this instance, the vertices that are not divisible by 3 (namely **1**, **2**, **4**, **5**, **7**, and **8**) are excluded from the second figure, just as odd values were excluded from the first.

Other examples of ***n***,***J***-stars with fewer than ***n***-points, triangular and otherwise, are easy to construct. All that is required is that ***n*** and ***J*** have a factor (larger than 1) in common.

When two numbers have a factor larger than 1 in common, there must be a largest such factor. There are two interchangeable terms for this factor: *greatest common factor* and *greatest common divisor*, GCD. Finding the GCD of two numbers is discussed elsewhere. [In *Excel*, type =GCD(***a***,***b***) to find the GCD between two numbers ***a*** and ***b***.]

VCF. The polygons and stars discussed here are an important component of the string art model. Indeed, they form the skeletal framework on which string art images are hung. We will fill in the rest of the model in the next chapter, but here we should acknowledge that because analyzing string art images depends on more than one GCD, we have a special name for GCD(***n***,***J***), namely the *Vertex Common Factor*, **VCF**.

VCF = GCD(***n***,***J***).

How many points does an *n*,*J*-star have? An ***n***,***J***-star will have ***n***/VCF-points. Put another way, if we examine the image of an ***n***/VCF,***J***/VCF-star, it would be the same as the ***n***,***J***-star (except that unused vertices would no longer be visible). Take a simple example. The 5,2-star, VCF = 1 at bottom left (pentagram), is the same image as the 10,4-star, VCF = 2 at bottom middle, or the 15,6-star, VCF = 3 at bottom right. The only difference is the fraction of vertices used in each case, 1/VCF, so the bottom left uses 100% of its vertices, bottom middle uses 50% and bottom right uses 1/3 of its vertices.

Fact. Every value of ***n*** > 4 EXCEPT for 6 has at least one ***J*** that produces an ***n***-point ***n***,***J***-star. This is true because one can always find at least one value of ***J***, 1 < ***J*** < ***n***/2 with GCD(***n***,***J***) = 1 as long as ***n*** > 6. Fewer ***J*** satisfy this criterion when ***n*** is small and composite and only one ***J*** satisfies this criterion when ***n*** = 8, 10, and 12 (namely, ***J*** = 3, 3, and 5, respectively).