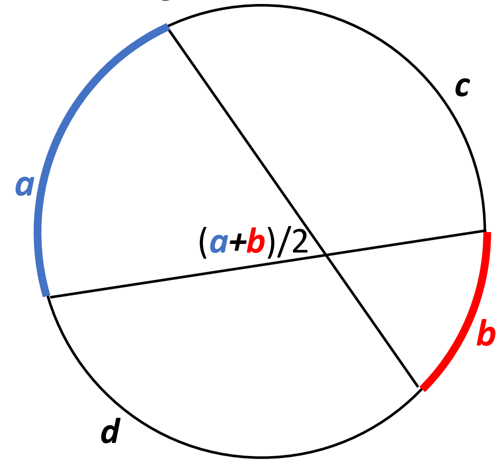
About Interior Angles and Parallel Lines

Two intersecting line segments (chords of a circle) create angles in one of two ways: *Inscribed Angles* and *Interior Angles.* Inscribed angles occur if the intersection is on the circle while interior angles occur if the intersection is on the interior of the segments.

Interior Angles. *Interior angles* are created when two segments intersect on the interior of the circle. Four angles are created but only two of them are distinct. If ***a*** and ***b*** represent the two opposing arcs of a circle, then the angle created is (***a***+***b***)/2 as shown in **I**. to the right. However, if ***a*** and ***b*** represent the number of vertices between regular ***n***-gon vertices then ***a***+***b***+***c***+***d*** = ***n***, and the angular measure of the interior angle is (***a***+***b***)·180/***n***°.

I. Interior angles and arcs of a circle



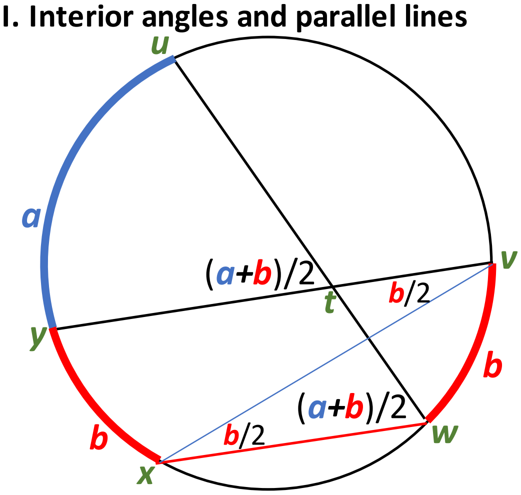
Note that had we examined the other two arcs created by these two lines, ***c*** and ***d***, then the measure of the angle created from arcs ***c*** and ***d*** is (***c***+***d***)/2 which is supplementary to (***a***+***b***)/2. *Supplementary angles* sum to 180°. Since ***a***+***b***+***c***+***d*** = 360°, (***a***+***b***)/2 + (***c***+***d***)/2 = 180°, if ***a***, ***b***, ***c***, and ***d*** are considered arcs of a circle. If ***a***, ***b***, ***c***, and ***d*** are the number of vertices between endpoints, the interior angles are 180·(***a***+***b***)/***n***° and 180·(***c***+***d***)/***n***°.

Interior angles can be thought of as a generalization of inscribed angles. If one of the arcs is of zero size (i.e., just a point) then we have an inscribed angle. And in this instance, the measure of the interior angle becomes the measure of the inscribed angle, half the size of the arc (or central angle).

Exploring why this formula works using parallel lines. As noted in the *Analyzing Stars inside a Star* explainer, parallel lines occur when lines are the same number of vertices different from one another on both sides. This is simply a discrete example of a more general notion about arcs of a circle.

The ***green labels*** in Figure **II** denote endpoints of arcs and the intersection point ***t*** for ease of referencing below. Start from **I**. and rotate a copy of the arc ***b*** clockwise around the circle until it just touches arc ***a*** at point ***y*** as shown in Figure **II**.

II. Interior angles and parallel lines



Draw a line (shown in **red**) connecting ***x*** and ***w***. The angle ***xwu*** is half the size of the arc ***a***+***b*** according to the *Inscribed Angle Theorem*.

Consider the light blue line from ***x*** to ***v***. This creates two angles ∠***yvx*** and ∠***vxw*** both of which measure ***b***/2 according to the inscribed angle theorem. These are *alternate internal angles* of the lines ***xw*** and ***yv*** meaning that these two lines are parallel.

Since the lines are parallel, corresponding angles are the same. Therefore, since ∠***xwu*** = (***a***+***b***)/2, we know that ∠***ytu*** = (***a***+***b***)/2. This discussion shows how the interior angle theorem and inscribed angle theorem are related.