On why the Angle Table has a Ragged, but Regular, Right Edge Pattern

Sharpest stars occur when ***J*** is the largest number less than ***n***/2. Given that, you might expect the progression of sharpest star ***J*** values to increase by 1 every time ***n*** increase by 2. Stylistically, you would expect the table to have the right-most cell (largest ***J***) for any ***n*** increase as shown by ■ in Table 1.

Expected Sharpest Star Pattern

Table 1	…	*J*	*J*+1	*J*+2	*J*+3	*J*+4
…						
n		■				
n+1		■				
n+2			■			
n+3			■			
n+4				■		
n+5				■		
n+6					■	
n+7					■	
n+8						■

Given, ***J*** < ***n***/2 but ***J***+*1* ≥ ***n***/2.

This table works best if you think of ***n*** as being odd so ***n*** = 2***k***+1 and ***n***/2 = ***k***+½ which means the largest ***J*** is ***J*** = ***k***. The next ***n***, ***n***+1 = 2***k***+2 so half that is ***k***+1 but this cannot be used for ***J*** as this creates a vertical line image. Therefore, the largest value of ***J*** possible given ***n***+1 if ***n*** was odd is ***J*** = ***k***. This is first stack of two ■ cells in Table 1. To make this concrete: if ***k*** = 3 then ***n*** = 7 = 2·**3**+1 and ***n***+1 = 8 both of these values of ***n*** have sharpest stars with ***J*** = 3. The next stack simply increases ***J*** by 1 as ***n*** has increased by 2 and so on.

If you look at the *Angle Table* and focus on the right edge, it does NOT step down in this fashion. Instead, it looks like Table 2. The obvious difference is the location of the sharpest ***n***+3 star which, rather than being at ***J***+1 is at ***J***.

Actual Sharpest Star Pattern

Table 2	…	*J*	*J*+1	*J*+2	*J*+3	*J*+4
…						
n		■				
n+1		■				
n+2			■			
n+3		■				
n+4				■		
n+5				■		
n+6					■	
n+7				■		
n+8						■

This issue is discussed in *Not all Even Stars are Created Equal*. The point is even sharpest stars alternate between having an angle that spans 2 and 4 vertices. That analysis focused attention on whether the even ***n*** was divisible by 2 but not 4 or by 4. We will come at the same issue in a slightly different way here.

Even *n*. An even ***n*** means that ***n*** = 2***k***. Choosing ***J*** = ***k*** produces a vertical line, so the sharpest possible alternative is ***J*** = ***k***-1.

CLAIM: This will produce an ***n***,***J***-star with ***n***-points only if ***J*** is odd.

J* odd.** If ***J = ***k***-1 is odd, then ***k*** is even or ***k*** = 2***h*** in which case ***n*** = 2·2***h*** = 4***h*** and hence ***n*** is divisible by 4. Such an ***n***,***J***-star would span two vertices.

J* even.** If ***J = ***k***-1 is even, then the ***n***,***J***-star will have ***n***/2 points because the odd vertices of the even ***n***-gon will be excluded from the final star via this jump pattern. As jumps of ***J*** vertices per line added are counted around the polygon, one will maintain only even endpoints even after one passes over the top of the ***n***-gon because **n** is even. To be explicit, consider the first three jumps (from 0 to ***J*** to 2***J*** to 3***J***).

0 to ***J*** ends at a vertex that is even by assumption.

J to 2***J*** is the same as ***k***-1 to 2***k***-2 = ***n***-2. Since ***n*** is even, so is ***n***-2.

2***J*** to 3***J*** is the same as 2***k***-2 to 3***k***-3 = ***n***+***k***-3 which ends at vertex ***k***-3. By assumption, ***J*** = ***k***-1 is even, so ***k***-3 is as well.

This process continues until all even vertices have been used and an ***n***/2,***J***/2-star results.

Since ***J*** = ***k***-1 does not produce an ***n***-point star, ***J*** = ***k***-2 will because ***k***-2 is odd, so ***k*** is odd, ***k*** = 2***h***+1 in which case ***n*** = 2·(2***h***+1) = 4***h***+2, or ***n*** is divisible by 2 but not 4. Such an ***n***,***J***-star would span four vertices.

To Conclude: The pattern of sharpest angle stars shown above starts from an odd ***n***, but more specifically from an ***n*** of the form ***n*** = 4***k***+3 with ***J*** = 2***k***+1. The next three sharpest stars (**n**, ***J***) values are (***n***+1, 2***k***+1), (***n***+2, 2***k***+2), and (***n***+3, 2***k***+1). This is easy to check for ***k*** = 1: the 7,3-star; 8,3-star; 9,4-star, and 10,3-star are all the sharpest stars for each of these values of ***n***.