

## Calculating Lines used given Jump Sets

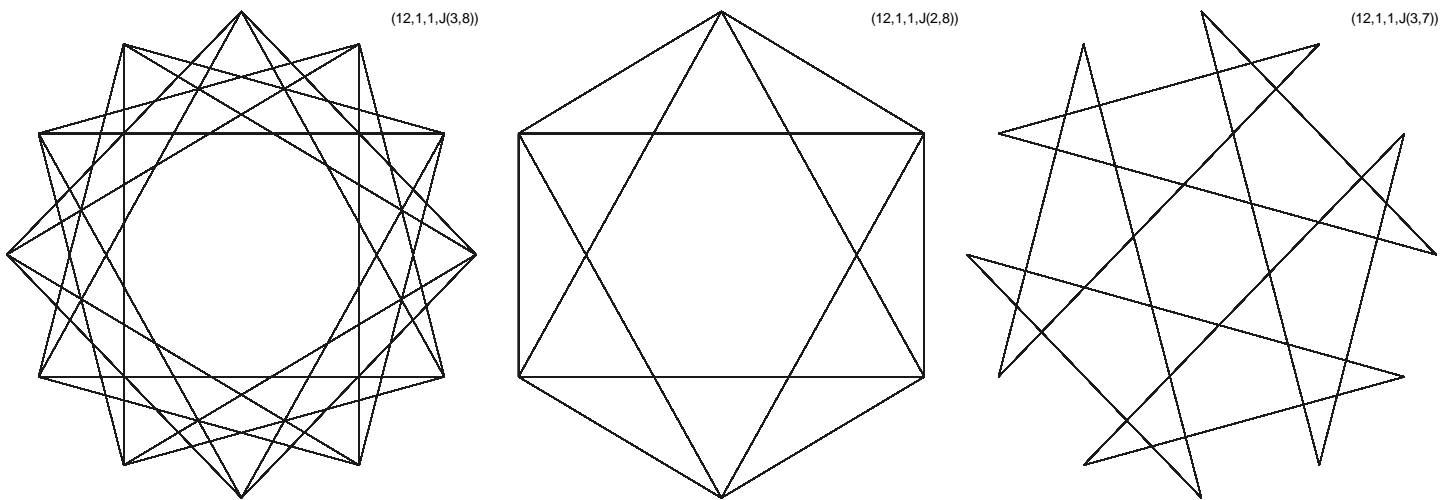
Stars are created from having a vertex jump of larger than 1, the most common is the pentagram obtained from  $n = 5$  and  $J = 2$ . The jump of 2 occurs repeatedly until the initial vertex is obtained. Jump sets work in the same fashion except two or more different jumps occur before the pattern is repeated. The image is completed once the initial vertex is achieved AT THE END OF A JUMP SET. **A jump set is a repeated pattern of different jumps.**

Images A-C involve  $S = P = 1$  (so each image is the vertex frame, VF) and  $n = 12$  (so that it is easy to talk about jumps in "hour" terms meaning that if you are at 11 o'clock and add 3 hours you get to 2 o'clock not 14). In Panel A,  $J_1 = 3, J_2 = 8$ , leads to jumps of:

The vertex jump pattern is thus:                    3, 8, 3, 8, 3, 8, **3**, 8, ... 3, 8  
**In hours**, the image is drawn as:                    12&0 to 3 to 11 to 2 to 10 to 1 to 9 to **12&0** to 8 ... to 4 to 12&0

Notice, in particular, that the pattern does NOT stop at the bolded **12&0** above because the jump of **3** from 9 is not the end of the jump sequence. The end of the sequence takes  $2n = 24$  jumps. Note that each vertex is used TWICE (there are 2 lines in and out of each vertex rather than 1). More generally, if there are  $k$  jumps in the sequence, the pattern of vertex usage will end after  $k \cdot n$  jumps if there is no commonality between  $n$  and the sum of the  $k$  jumps. (The next chapter focuses on a four-color *Excel* file that allows up to 3 jumps for  $n = 12$  and the final jump-set chapter allows a larger number of jumps in the set.) In Panel A,  $3+8 = 11$  has no divisor in common with 12 but in Panels B and C,  $2+8 = 10$  and  $3+7 = 10$  both have 2 in common with 12 therefore half as many jumps occur before completing the VF. Both are shown because they achieve this in different ways: B uses half the vertices, 6, twice each, and C uses all vertices, 12, once each.

**About labels.** Since jump sets are open-ended, they are denoted by  $J(\#, \dots, \#)$  at the end of the label (for example,  $J(3,8)$ ).



A) All vertices used twice, VCF = 1                    24 lines in VF                    B) Half the vertices used twice, VCF = 2                    12 lines in VF                    C) All vertices used once, VCF = 2                    12 lines in VF

**Calculating VCF.** The equation for VCF must be altered given jump sets. Then VCF is:

Given  $k$  jumps per set, the *Vertex Common Factor*, VCF, is:                     $VCF = \text{GCD}(m, n)$ , where  $m = J_1 + \dots + J_k$  and GCD is the greatest common divisor function.

The number of vertices used,  $v_{used}$ , (*perhaps multiple times*) and lines in VF is:                     $v_{used} = k \cdot n / VCF$ .

**SCF.** The number of VF line segments connecting used vertices,  $v_{used}$ , changes with jump sets but we still create  $S$  subdivisions on each VF line leading to  $S \cdot v_{used}$  possible subdivision endpoints. Nothing changes to SCF as a result of jump sets, it is still  $SCF = \text{GCD}(P, S \cdot v_{used})$  and therefore nothing changes to lines used,  $L = S \cdot v_{used} / SCF$  (except  $v_{used}$  is larger).

**Subdivision size differences.** In contrast to the traditional string art model, subdivision size is not constant across jumps. The largest difference in VF segment size is between  $J = 1$  and  $J = \text{INT}(n/2)$ , but both have  $S$  subdivisions per segment.

**NOTE:** The number of lines in the vertex frame may appear less than  $v_{used}$  because some lines are used multiple times.