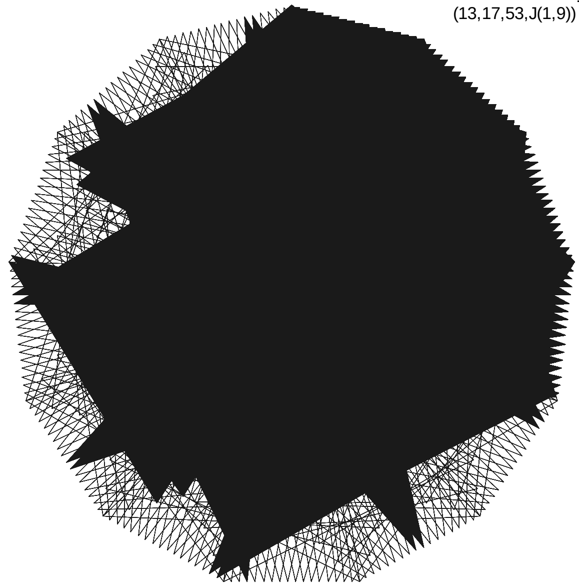
Revisiting *Single-Step* Images



(13,17,53,J(1,9))

One of the images highlighted in Visualizing VCF, SCF, and Image Density is shown to the right. **A.** What is it about the image that makes it single-step? **B.** What can be changed while maintaining its single-step status? These are the questions we examine here.

Answering Question A. Consider the *first* single-step**.** As noted there, the 25th line ends near vertex 0 on the line from vertex 4 to vertex 0. This is a point just below the top of the image, as can be seen using *Single Line Drawing, SLD,* mode on *Pause* with *Drawn Lines, DL* = 25 and *Drawing progress* = 0. Note the following four points:

1) Subdivisions counted at that point: 25·53 = 1325.
2) INTEGER(1325/17) = 77, remainder MOD(1325,17) = 16 meaning that this endpoint is 1 subdivision from the 78th VF line. Put another way, 78·17 = 1326, one more than 1325.
3) Every 2 VF lines is a jump of 10, so 39 (39 = 78/2) jumps of 10 vertices, or 390 total vertices jumped.
4) 390 vertex jumps end at vertex 0 = MOD(390,13) given ***n*** = 13.

This is why the first single-step ends where it does, within 1 subdivision of the top of the image.

[**MA**. 25 is the negative MMI of ***P*** MOD ***S*** in addition to (J_1+J_2)·(INTEGER(25·***P***/***S***)+1) = 0 MOD ***n*** since 25 is a negative MMI. Had it been a positive MMI, the second equation would not have added 1 since the subdivision endpoint would in that event be "just over" rather than "just under" the VF vertex in question.]

Next, consider static image creation and how to make the 1st single-step as small as possible before turning to **B**.

There are four ways to create this static image. By switching J_1 and J_2 one obtains the same static image. Additionally, by replacing both J_1 by ***n***-J_1 and J_2 by ***n***-J_2, the same static image results. So, for the above image, **J(1,9)** looks the same as **J(9,1)**, **J(12,4)** and **J(4,12)**. Four versions exist because four lines are attached to the top vertex in this instance, not two.

About the size of single-steps. Each line on the VF has ***S*** equally-sized subdivisions. But in jump set models, the lines are different sizes. The smaller the length of VF line, the smaller the size of subdivisions on that line. As a result, we can obtain the same static image, but have it drawn so that the first step is the smallest size possible by having the second jump be the smaller jump given the above values of ***n***, ***S***, and ***P***. When viewed in terms of how the outer edges is filled in, **J(9,1) turns ↺** while **J(4,12) turns ↻**. (Note that because ***n*** = 13, a jump of 12 has the same size VF line as a jump of 1.)

Answering Question B. What parameters can we relax while maintaining single-step status? We see that each of the parameters (***n***, ***S***, ***P***, J_1 and J_2) gets involved in creating this as a single-step image. But that is an illusion and there is greater flexibility here than one might imagine while keeping images single-step. One can quickly confirm with *SLD* mode using *DL* = 25 as discussed above, that altering ***n***, ***S***, and ***P*** voids single-step status.

One might expect that, due to points 2 and 3 above, as long as J_1+J_2 = 10, the image remains single-step. Indeed, this is true. But this works even when the sum is not 10 (or 16 = 2***n*** – 10). Indeed, *you obtain a single-step image whatever values you choose for both jumps***.** The reason why is based on how 39 (which is 78/2 recalling that that the 25th endpoint is 1 subdivision less than a multiple of 17 which denotes ends of VF lines) relates to ***n*** = 13. 39 is a multiple of 13 so a multiple of 39 is also a multiple of 13. Below are links to sample images. The bottom row is best viewed in *SLD*.

13,5-Star with spikes   11 Rotating Polygons   Rotating Pentagrams   Interior 26-point Sunburst   Ragged Flower Petals

Rotating Triangular Porcupine SLD   Rotating Bug I. *SLD*   Rotating Bug II. *SLD*   Swirling Moth *SLD*

Extensions. One can alter ***n*** and maintain single-step status by having ***n*** be divisible by 39 so here are *Rotating Triangular Porcupines* at ***n* = 26** and ***n* = 39** given *DL* = 25. Additionally, it does not matter which of the two negative MMI values is associated with ***P*** and which with *DL*. You can switch and have single-step images like this ***P* = 25 and *DL* = 53 *SLD* Crab**.