

## A Deeper Dive into Spiked Images

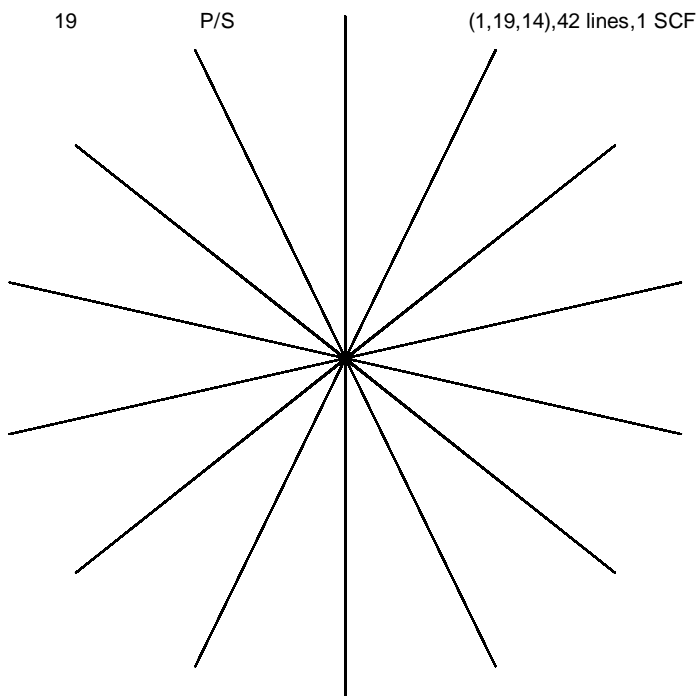
If you poke around CPF a bit more, you will find that you CAN get  $n$ -spikes for even  $n$  HALF THE TIME. More specifically, if  $n$  is divisible by 2 but not 4, then  $m = 3/2n - 2$  produces  $n$  spikes, but if  $n$  is divisible by 4,  $n/2$  spikes are produced. The table for  $n = 12$  and  $m = 16$  found in the [P = 2S explainer](#) will be useful as a contrast to the discussion below).

**Automating the CPF file.** If you type  $=1.5*L1-2$  in C1 for  $P$  and set  $S = 1$ , then you can see spiked images for every even  $n$  created by  $m > 2$  simply using the  $\blacklozenge$  for  $n$ . A | line occurs for  $n = 4$ ,  $n = 6$  has the same  $*$  image as  $n = 12$ , and  $n = 8$  is a +. We focus here on the largest  $n$  which is divisible by 2 but not 4 available in the CPF file,  $n = 14$ . The resulting 42-line image is below. The table shows how this image was created (note the 43<sup>rd</sup> and 1<sup>st</sup> Lines share Start and End vertices).

**Modifying the table.** To deal with general  $n$ , create a cell for  $n$  (add a new row and put it in B1), replace  $m$  with  $=1.5*B1-2$  in B2, and replace 12 by  $n$  in MOD  $n$  vertices equations).

**The pattern is a double jump backwards.** The 1<sup>st</sup> line ends at  $n/2$  and the 3<sup>rd</sup> line ends 2 less than  $n/2$ , just like with  $n = 12$  (where the 1<sup>st</sup> line ended at 6 and the 3<sup>rd</sup> line ended at 4). In this instance, the 1<sup>st</sup> line ends at 7 and the 3<sup>rd</sup> ends at 5. The second cycle moves half-way around but does the same reduction by 2 vertices in moving from 4<sup>th</sup> to 6<sup>th</sup> lines (from 10 to 8 given  $n = 12$  or 12 to 10 for  $n = 14$ ). After this, the third cycles first ending (at Line 7) is 2 less than where the first cycle ended (at 2 for  $n = 12$  or 3 for  $n = 14$ ).

In both instances, successive jumps backward of 2 are the norm. The difference is when  $n = 4k$ , the values that are backing up to 0 are all even so it stops at 0. If  $n = 4k+2$ , these same vertices are odd ( $n/2 = 2k+1$ ) and backing up 2 from 1 yields 13 in the move from Line 9 to 13. At this same time, even vertices begin to be filled in on the right half of the circle (Line 10 ends at  $8 > n/2$  but Line 12 ends at  $6 < n/2$ ). This is why all vertices are used when  $n = 4k+2$ , but only half are used when  $n = 4k$ .



Line, L	P = mL*	Vertex number of n-gon or Center, C		Vertices (MOD n)		Diameter or radius
		Start	End	Start	End	
1	19	0	7	0	7	D
2	38	7	C	7	C	R
3	57	C	19	C	5	R
4	76	19	26	5	12	D
5	95	26	C	12	C	R
6	114	C	38	C	10	R
7	133	38	45	10	3	D
8	152	45	C	3	C	R
9	171	C	57	C	1	R
10	190	57	64	1	8	D
11	209	64	C	8	C	R
12	228	C	76	C	6	R
13	247	76	83	6	13	D
14	266	83	C	13	C	R
15	285	C	95	C	11	R
16	304	95	102	11	4	D
17	323	102	C	4	C	R
18	342	C	114	C	2	R
19	361	114	121	2	9	D
20	380	121	C	9	C	R
21	399	C	133	C	7	R
22	418	133	140	7	0	D
23	437	140	C	0	C	R
24	456	C	152	C	12	R
25	475	152	159	12	5	D
26	494	159	C	5	C	R
27	513	C	171	C	3	R
28	532	171	178	3	10	D
29	551	178	C	10	C	R
30	570	C	190	C	8	R
31	589	190	197	8	1	D
32	608	197	C	1	C	R
33	627	C	209	C	13	R
34	646	209	216	13	6	D
35	665	216	C	6	C	R
36	684	C	228	C	4	R
37	703	228	235	4	11	D
38	722	235	C	11	C	R
39	741	C	247	C	9	R
40	760	247	254	9	2	D
41	779	254	C	2	C	R
42	798	C	266	C	0	R
43	817	266	273	0	7	D
End		IF(2=MOD(P,3),"C",IF(1=MOD(P,3),(P+2)/3,P/3))				
MOD n Start		IF(COUNT(Start)=1,MOD(Start,n),"C")				
MOD n End		IF(COUNT(End)=1,MOD(End,n),"C")				