## An Excel Primer on Double Jump Models (while achieving the elusive 6,2-star)

Background. The only $\boldsymbol{n}>4$ for which a continuously-drawn star cannot be drawn is $\boldsymbol{n}=6$. The problem is that a single-jump pattern move connecting every other vertex either attains the even vertices (0-2-4-0) or odd vertices (1-3-5-1) but not both.

We use $\boldsymbol{n}, \boldsymbol{J}$-stars to create the vertex frame for string art models. The VF forms the structure on which $\boldsymbol{S}$ and $\boldsymbol{P}$ depend to create the final image. To understand what happens when we move from a singlejump pattern model to a multiple jump pattern model, it is worthwhile to initially focus on the VF in this new environment. Thus, the image to the right has $\boldsymbol{S}=\boldsymbol{P}=1$. As this image attests, if we allow a double-jump set pattern, we can produce the 6,2-star, albeit with diameter lines included in the image. Those diameter lines allow even numbers to intersperse with odd numbers in the overall pattern.

Definition. A jump set is a pattern of jumps that is repeated. The traditional string art model was a single-jump pattern (although it was not discussed that way to avoid confusion because we do not mean
 that $\boldsymbol{J}=1$ ). We consider a double jump model where a pattern of two different jumps is repeated, in particular $\boldsymbol{J}_{1}=2$ and $\boldsymbol{J}_{2}=3$ with $\boldsymbol{n}=6$. The initial four jumps (two sets) are 0-2-5-1-4. Note that this creates a setting in which even and odd vertices are interspersed. The final image is completed in 12 jumps.

Even if you do not write out all the jumps, you should try to visualize the pattern that created the image above. In this instance, it is quite simple: each jump of 2 is followed by a move to the opposite side of the $n$-gon since, in this instance, $3=6 / 2$ so that a jump of 3 is simply a move from one end a diameter to the other. (Recall, a 6,3-star is a vertical line.)

If you had been asked to follow this jump pattern with another value of $n$, it would not be quite as easy to follow as it is easy to forget which part of the jump set you are in. You can avoid this issue by creating an Excel file to do the work for you. And, if you want to change the underlying values of $n, \boldsymbol{J}_{1}$ or $\boldsymbol{J}_{2}$ just type in different numbers for these parameters.

Using Excel to find the VF. The table at left below shows how this is done (in 2 parts). To create your own table, create the 4 left columns using the instructions provided in the right half. This involves 10 cells: 4 numbers (highlighted in green), 3 equations (in crème), and 4 equations that will be dragged (in yellow). (The sideways sum in C5 is a useful trick.) Column D shows the VF jump pattern. Importantly, note that Line 5 is the $1^{\text {st }}$ time 0 is an endpoint vertex but the circuit is not complete because it was attained by a $\boldsymbol{J}_{\mathbf{1}}$ jump not a $\boldsymbol{J}_{\mathbf{2}}$ jump like Line 12.

|  |  |  |  | 4 | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 Jump 1 |  |  | 6 | 1 | 2 Jump 1 |  |  | 6 |
|  | Jump 2 | Size of $\boldsymbol{n}$-gon |  | 2 | 3 Jump 2 |  | (Sideways) | Size of $\boldsymbol{n}$-gon |
| Line | Pattern | Sum | Vertex | 3 | Line | Pattern | Sum | Vertex |
| 1 | 2 | 2 | 2 | 4 | 1 | =A1 | =B4 | =MOD(C4,D\$1) |
| 2 | 3 | 5 | 5 | 5 | =A4+1 | =A2 | = $\mathrm{C} 4+\mathrm{B5}$ | drag D4 down |
| 3 | 2 | 7 | 1 | 6 | drag | =A4 | drag C5 |  |
| 4 | 3 | 10 | 4 | 7 | A5 | drag | down |  |
| 5 | 2 | 12 | 0 | 8 | down | B6 | Highlighted | cell color scheme |
| 6 | 3 | 15 | 3 | 9 |  | down | Ente | numbers |
| 7 | 2 | 17 | 5 | 10 |  |  | Single u | e equations. |
| 8 | 3 | 20 | 2 | 11 | Equat | ns to | ag down as | $r$ as necessary to |
| 9 | 2 | 22 | 4 | 12 | see the | pattern | repeat (as w | e see at Line 13). |
| 10 | 3 | 25 | 1 | 13 | Expla | anation of | of equation in | cell D4 above |
| 11 | 2 | 27 | 3 | 14 | MOD is | the rem | mainder funct | on when one |
| 12 | 3 | 30 | 0 | 15 | numbe | $r$ (C4, he | ere 2) is divid | d by another |
| 13 | 2 | 32 | 2 | 16 | (D\$1, h | ere 6). Th | The $\mathbf{1} \mathbf{1}$ in this | Excel equation |
| 14 | 3 | 35 | 5 | 17 | means | fix this in | in Row 1 upon | dragging down. |

The table to the right and the image below show that, by changing 3 numbers, we see how this jump pattern creates superimposed 11,3and 11,6-stars.


| $\begin{aligned} & 3 \text { Jump } 1 \\ & 6 \text { Jump } 2 \end{aligned}$ |  |  | 11 |
| :---: | :---: | :---: | :---: |
|  |  | Size of $\boldsymbol{n}$-gon |  |
| Line | Pattern | Sum | Vertex |
| 1 | 3 | 3 | 3 |
| 2 | 6 | 9 | 9 |
| 3 | 3 | 12 | 1 |
| 4 | 6 | 18 | 7 |
| 5 | 3 | 21 | 10 |
| 6 | 6 | 27 | 5 |
| 7 | 3 | 30 | 8 |
| 8 | 6 | 36 | 3 |
| 9 | 3 | 39 | 6 |
| 10 | 6 | 45 | 1 |
| 11 | 3 | 48 | 4 |
| 12 | 6 | 54 | 10 |
| 13 | 3 | 57 | 2 |
| 14 | 6 | 63 | 8 |
| 15 | 3 | 66 | 0 |
| 16 | 6 | 72 | 6 |
| 17 | 3 | 75 | 9 |
| 18 | 6 | 81 | 4 |
| 19 | 3 | 84 | 7 |
| 20 | 6 | 90 | 2 |
| 21 | 3 | 93 | 5 |
| 22 | 6 | 99 | 0 |

