

## A Visual Interpretation of VCF and SCF

**Background.** The 12 images on the next page share a common double jump set pattern of  $J_1 = 1, J_2 = 9$ . Values for other parameters were chosen to highlight difference that arise as VCF and SCF vary.

The images are set up in three rows and four columns. Each row is a different value for  $n$ . The first column shows the vertex frame in each instance. The last three columns share a common  $S = 17$  and each column has a different  $P$ , the first two of which have  $SCF = 1$  and the final column has  $SCF > 1$ .

**Vertex Frame.** The *Excel* file discussed in the [Vertex Frame Primer with Excel](#) is able to examine more than one  $n$  at a time simply by dragging the 4<sup>th</sup> column to the right (after making the adjustment described in the **MA** note) and changing  $n$  as necessary.

[**MA.** The equation in cell D4 needs to be slightly adjusted to be dragged sideways,  $=MOD(C4,D\$1)$  needs have a \$ sign added to C4 so that it does NOT turn into D4 when dragged sideways. This is accomplished by fixing column using a \$ sign in front of the C. The modified equation is  $=MOD(\$C4,D\$1)$ . When dragged sideways, C remains fixed, and D turns into E. This is formally called *Mixed Referencing* in *Excel* and it is a very powerful tool to learn how to use.]

The three values of  $n$  examined here are 12, 13 and 15 as can be seen in the table in cells D1, E1 and F1. In each column, highlighted cells are repeats from above since the circuit has been completed (at Lines 12, 26, and 6 by column).

**VCF.** If  $VCF = 1$ , then the VF has  $2n$  lines, but more generally, the VF has  $2n/VCF$  lines since  $k = 2$  for the double jump model as discussed in [Calculating Lines](#). Both examples in the *VF primer* had  $VCF = 1$  but here we show images with  $VCF > 1$ , similar to those shown in *Calculating Lines*. As can be seen the first column of images,  $VCF = 2$  for  $n = 12$ ,  $VCF = 1$  for  $n = 13$ , and  $VCF = 5$  for  $n = 15$ , consistent with the circuit lengths shown in the table to the right.

The sum of jumps,  $J_1 + J_2 = 10$ , so any even  $n$  will have  $VCF \geq 2$ . If you check other even  $n$  such as  $n = 14$  you will find  $VCF = 2$  so 14 vertices are used. Note also that  $VCF = 10$  when  $n = 20$  and 4 vertices are used (the VF would be 0-1-10-11-0).

**SCF.** Prime values were chosen for  $P$  in the middle two columns (11 and 53) to avoid  $SCF > 1$  drop out of lines. Line usage ([image density](#)) drops off by a factor of  $SCF$  (4, 26, and 2) when  $P$  decreases by 1 to  $P = 52$  in the final column.

**Locating the first line in the 3<sup>rd</sup> and 4<sup>th</sup> columns.** Since  $51 = 17 \cdot 3$ , we know that the subdivision endpoint of the first line is just past the end of the 3<sup>rd</sup> line on the 4<sup>th</sup> line of the VF (2 subdivisions in in the 3<sup>rd</sup> column and 1 subdivision in in the second). Look at the VF in the first column and count the first 3 lines out to see where that point is likely to be.

**Drawing Mode.** To check your work here, or to watch the images get drawn, it is useful to use the web version of each image. Rather than provide 12 different links, note that once you have one, the others are easy to get to by adjusting  $n$  and  $P$  since  $S, J_1$  and  $J_2$  are fixed for all 12 images. Perhaps the best is the [third column middle row](#) which, if you set *Drawn Lines* = 25 using the *Fixed Count Drawing Mode*, you will see this image emerge as an interesting example of [Single Step Images](#). The 25<sup>th</sup> line ends on a section of the VF that is a 9-jump (from 4 to 0 =  $4+9 = 13 \text{ MOD } 13$ ) and subdivisions on such segments are much larger than those on 1-jump segments (use *Pause/Play* to verify location).

**Similar images for different  $n$ .** When  $P = 11, n = 14$  and 16 versions look like  $n = 12$  for an internal heptagon (7) or octagon (8) in place of the hexagon. The  $P = 53$  version for those two  $n$  appear similar to the  $n = 15$  versions except now there are 7 or 8 locations for the "Palm fans" rather than 3 with  $n = 15$ . These three image have quite a different drawing experience. Check by setting *Drawn Lines* = 2 with *Drawing Speed* at 10 or so. Also, [n = 30](#) is a [needle fan](#).

1	J1		12	13	15
9	J2		Size of $n$ -gon		
Line	Pattern	Sum	vertices in $n$ -gons		
1	1	1	1	1	1
2	9	10	10	10	10
3	1	11	11	11	11
4	9	20	8	7	5
5	1	21	9	8	6
6	9	30	6	4	0
7	1	31	7	5	1
8	9	40	4	1	10
9	1	41	5	2	11
10	9	50	2	11	5
11	1	51	3	12	6
12	9	60	0	8	0
13	1	61	1	9	1
14	9	70	10	5	10
15	1	71	11	6	11
16	9	80	8	2	5
17	1	81	9	3	6
18	9	90	6	12	0
19	1	91	7	0	1
20	9	100	4	9	10
21	1	101	5	10	11
22	9	110	2	6	5
23	1	111	3	7	6
24	9	120	0	3	0
25	1	121	1	4	1
26	9	130	10	0	10
27	1	131	11	1	11

