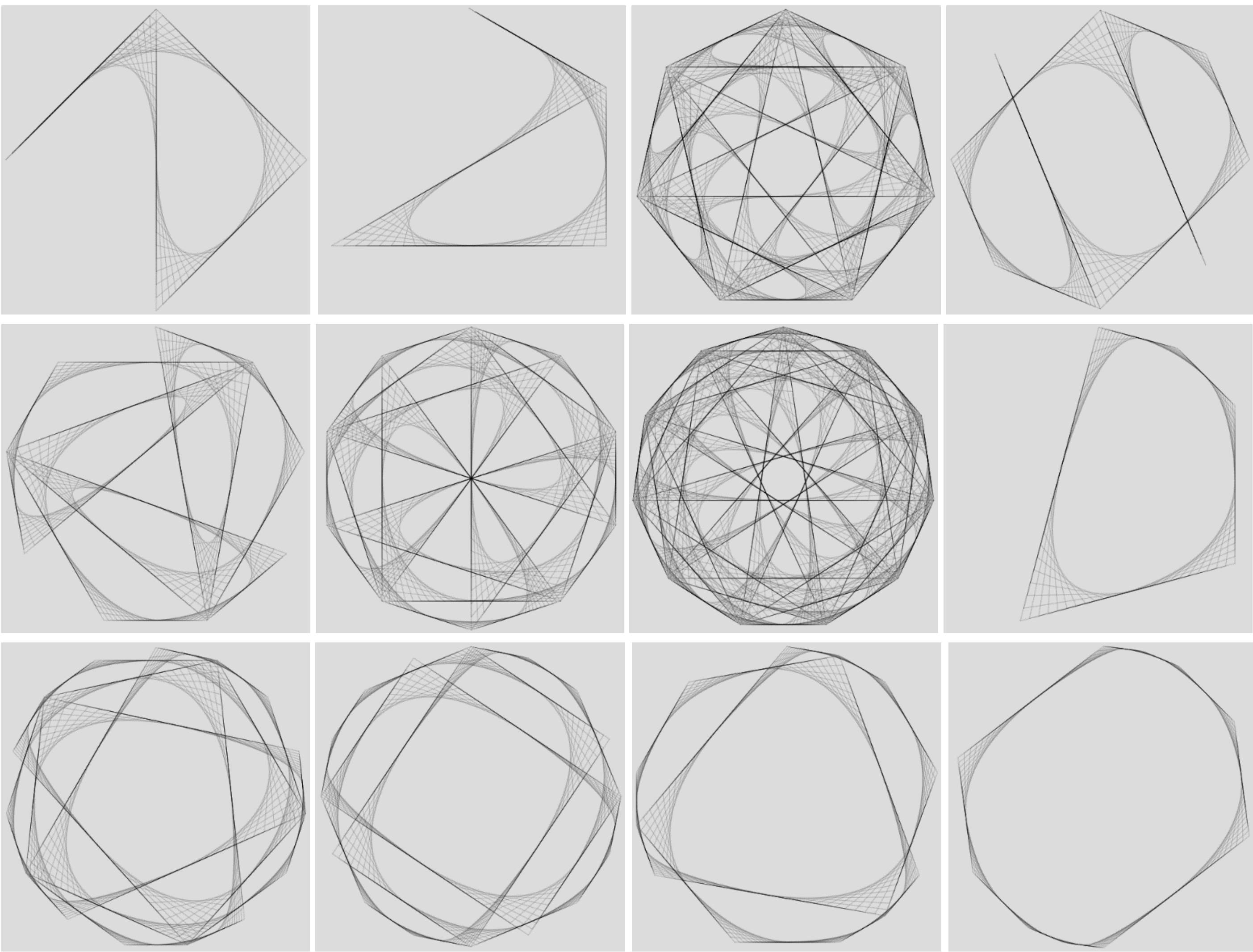
Fibonacci

Introduction. If you have not run into Fibonacci numbers before, it is worth a quick Google search. The ubiquity of this sequence suggested that it would be worthwhile to see if creating a jump set of Fibonacci numbers would yield interesting images. Indeed, they do.

The Fibonacci Sequence is simple: The k^{th} Fibonacci number is the sum of the previous two Fibonacci numbers. The first 10 numbers in the sequence thus goes: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, … . For simplicity, call the k^{th} value $\mathbf{F}_k$.

One of the interesting properties of Fibonacci numbers is that the sum of the first ***k*** Fibonacci numbers is 1 less than $\mathbf{F}_{k+2}$. It is easier to get the sum by looking later in the sequence than by repeated addition of individual Fibonacci numbers.

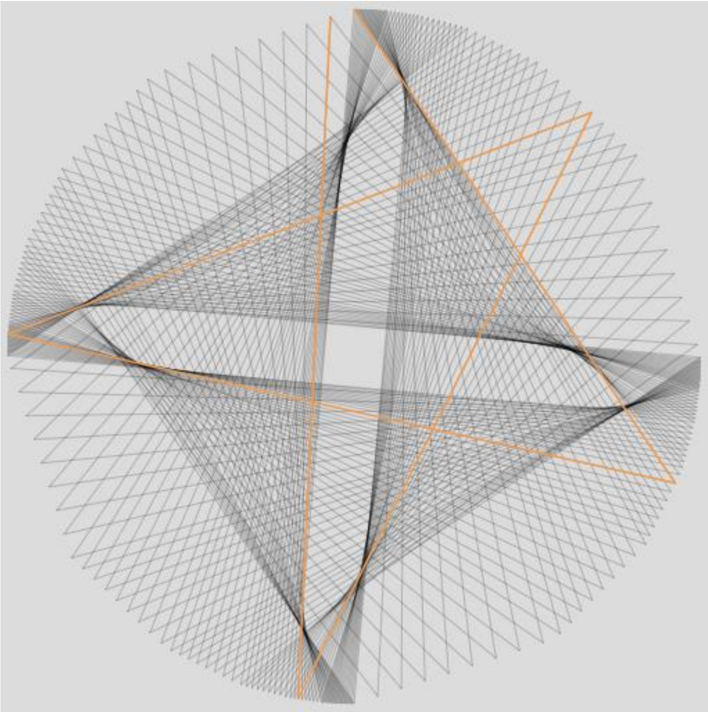
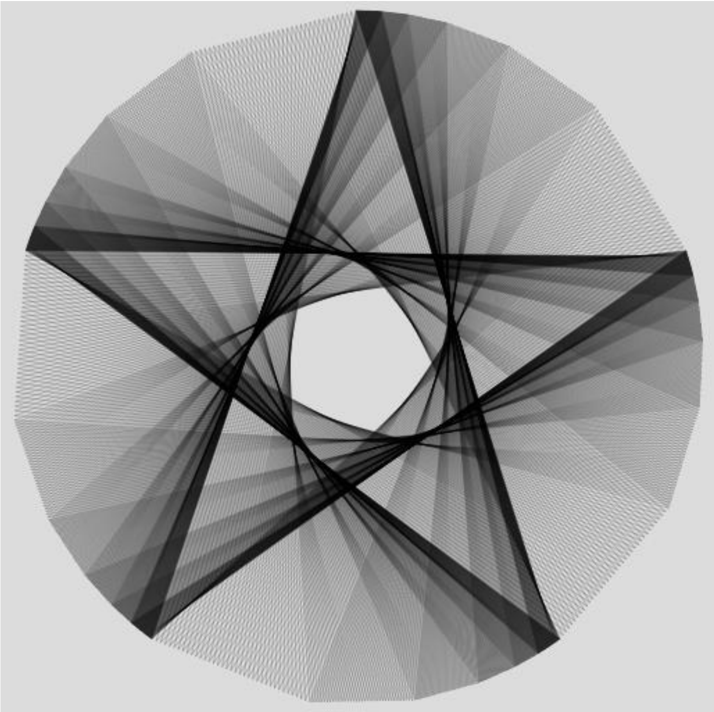
Curved-tip stars given $\mathbf{F}_5$. The first 5 Fibonacci numbers sum to 12 so VCF = GCD(Sum of ***J***,***n***) = GCD(12,***n***). More interesting images are possible given a larger ***P*** but if ***P*** is close to half of ***S*** (here 13 and 23) the curves do not overlap much in the middle and it is easy to follow the pattern of curves through a jump set. Twelve images are shown in three rows from L to R: ***n*** = 4, 6, 7, and 8; 9, 10, 11, and 12; 15, 16, 18, and 24. The only ones with VCF = 1 are 7 and 11.



It is worthwhile to spend a few minutes looking at the images to make sure you can follow the first jump set: 1, 1, 2, 3, 5. In particular, you should be able to see why the first two images and ***n*** = 12 end after one jump set, the ***n*** = 8 and 24 images end after 2 sets, the ***n*** = 9 and 18 images end after 3 jump sets, and why three of the images have spikes.

Many of these images have inherent spirals, which is not surprising given that the jump sequences are Fibonacci. A well-known result is that you can approximate a golden spiral using Fibonacci numbers.

When working with larger Fibonacci sequences, you can end up with interesting images when VCF > 1. This infinity symbol image is based on ***n* = 9 and $\mathbf{F}_8$**. The sum of jumps is 54 and the image is completed in a single 8 jump set.

Top left image, shown with first 5 lines drawn, is **F_7** swirling single-step pentagram, DL = 5. The image has 90° rotational symmetry because ***n*** = 132 or 4 times the sum of the first 7 Fibonacci numbers. This porcupine pentagram needle fan, DL = 2 has ***n*** = 165 = 5·33 is a close cousin as a result.

The top right pentagram with creases is another ***n*** = 165, **F_7** image. The creases are created by having the first 7 Fibonacci numbers create different densities of lines as vertices in the 165-gon are traversed 5 times. Note: the five least dense sets of lines are 13/165 = 7.8% of the circle or a bit less than 1/12 = 8.3% (at about hours 2, 4, 7, 9 and 11).

Double Fibonacci. The bottom left image is **F_7** up then back down (or 14 jumps in a set with sum of jumps = 66) has 1036 lines with VCF = 22. This image has ***n*** = 44. Here are additional images with the only difference being alternative ***P*** values: 53 61 75 223 359 423 Letter **O** Diamond 503 and Porcupine 517. If you return to the bottom left image but change ***n***, you get a scowling face for ***n*** = 6. Also consider checking ***n*** = 18, 22, 24, 30, 33, 55, 66, 88, 99, 132, and 198.

The ***n*** = 13 bird beak has a jump set of 13 not 14. This allows VCF = 13 not 1. The ***J*** = 0 beak is created from $J_7 = J_8 = n$.