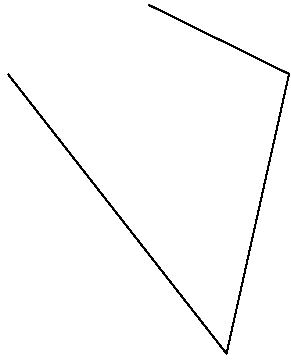
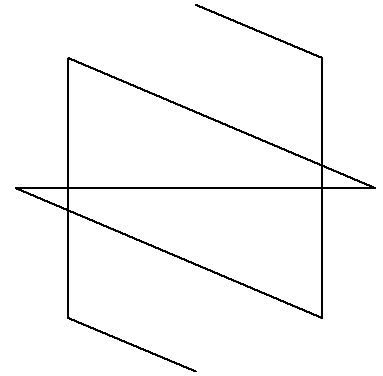


# Just Tails, Asymmetric Suspension Curves, and Spirals



How VF Tails are Made						Notes: $n-1$ jumps per set.								
7 $n$ When $n$ is odd						Increase 1 per jump								
Pattern	1	2	3	4	5	6	Results:							
Sum	1	3	6	10	15	21	Odd $n$ : 1 jump set to 0							
MOD $n$	1	3	6	3	1	0	Middle at $(n-1)/2$							
MOD $n$ is the vertex endpoint of points on the VF line.						Even $n$ : 2 jump sets to 0								
						Middle at $n-1$								
8 $n$ When $n$ is even						BOTH: Mirror about Middle								
Pattern	1	2	3	4	5	6	7	1	2	3	4	5	6	7
Sum	1	3	6	10	15	21	28	29	31	34	38	43	49	56
MOD $n$	1	3	6	2	7	5	4	5	7	2	6	3	1	0



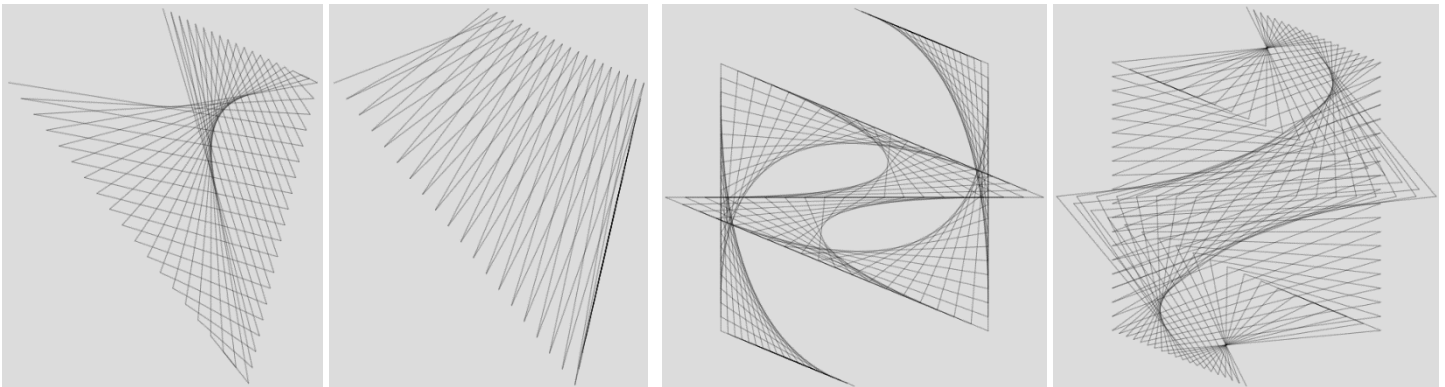
The table provides jump set notes for both images. The left image is  $n = 7$ , the right is  $n = 8$  and both are VFs. Both were created by following this rule.

**Just Tails Jump Rule:** Set  $J_i = i$  for  $1 \leq i < n$ . The table shows the jump pattern for both images using this rule.

**Claim:** This rule creates an  $n-1$  line VF with  $(n-3)/2$  kinks if  $n$  is odd, and a  $2(n-1)$  line VF with  $n-2$  kinks if  $n$  is even.

**Explanation.** The table shows the jump pattern for both images and provides the rationale for why each VF is simply a kinked series of lines. Note in particular the mirroring that occurs about the *Middle* of each VF. The reason for this is straightforward. The general jump pattern is 1, 2, 3, ...,  $n-3$ ,  $n-2$ ,  $n-1$ . The first and last sum to  $n$ , the 2<sup>nd</sup> and 2<sup>nd</sup> to last sum to  $n$ , and so forth. *Each jump in the second half, reverses the order of the jumps that were done in the first half.*

These images have  $S = 19$  and  $VCF = 1$ . From left to right  $P = 35$  and  $P = 55$  have 114 lines,  $P = 17$  and  $P = 83$  have 266. As is readily apparent, odd  $n$  images not rotationally symmetric but the even ones are. Turn them over, they look the same.

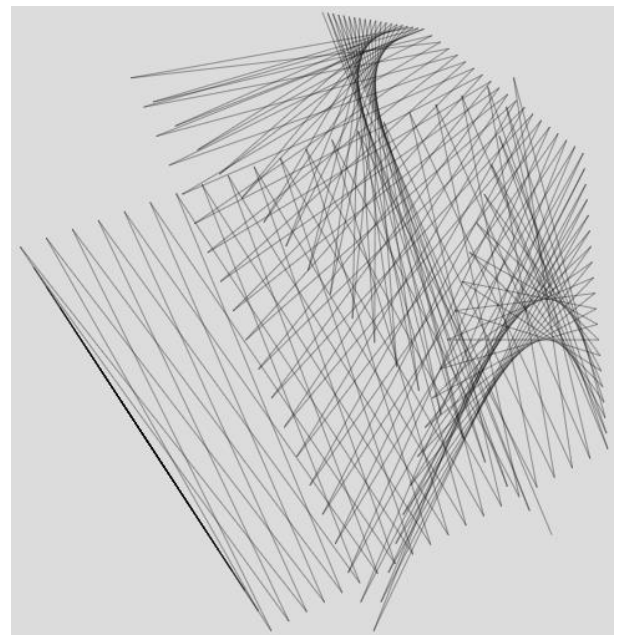


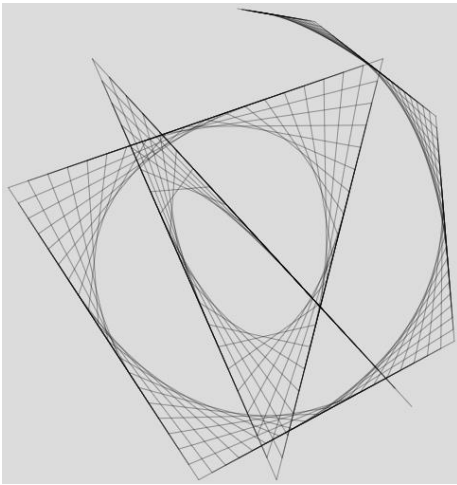
**Near Porcupines.** The second image is a near porcupine (2 less than halfway, 1 less has  $VCF > 1$ ). Even near porcupines are typically just very spiky images, but odd ones appear more interesting, for example  $n = 9$ ,  $n = 13$ , or this  $n = 19$ ,  $P = 169$  image.

**Asymmetric suspension curves, ASC.** In contrast to [suspension curves](#) found in single jump models, these two suspension curves are asymmetric because VF lines are of different lengths. The 342 line image is a near porcupine. If you increase  $P$  by 1 to 170, the curves become closer together, but the image has half the line density because  $VCF = 2$ . On the other hand, decreasing  $P$  creates greater distance between curves as witnessed by the  $VCF = 1$   $P$  values of 167 and 163.

ASC works for other odd  $n$ . The 418 line  $n = 23$ ,  $P = 207$  version is nice because as you decrease  $P$  by 2 from there,  $VCF = 1$  and the curves diverge. This 798 line  $n = 43$  version has 5 such curves.

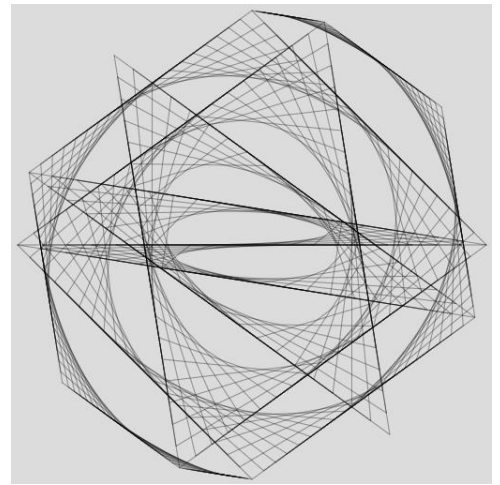
ASC also occurs without *just tails*, check out [this pentagonal one](#).





**Spirals.** Once  $n$  becomes sufficiently large, you can see that the *Just Tails Jump Rule* creates clockwise spirals if  $S > P$ . These four are  $S = 19 > 13 = P$ .  $n = 19$  is at left and  $n = 20$  is at right.

One vertex larger looks like a lot more rotations in the spiral, but that is because an odd  $n$  creates single spirals (odd *just tails* images are not rotationally symmetric) while an even  $n$  results in double spirals (because they are rotationally symmetric).



**How many rotations are in the spiral?** This depends on whether  $n$  is even or odd. For both even and odd, the image is created in two halves with the second half backing out of the first half. Therefore, we need only consider the first half.

**Odd  $n$ .** The first half of the jumps is  $(n-1)/2$  jumps. The sum of jumps is  $((n-1)/2+1) \cdot (n-1)/2 = (n-1)(n+1)/8 = (n^2 - 1)/8 = n^2/8 - 1/8$ , which, when divided by  $n$  is just below  $n/8$ .

*Examples:* When  $n = 7$ ,  $(n^2 - 1)/8 = (49-1)/8 = 6$ , just like the Jump 3 Sum in the top half of the table. This is 6/7 of a turn around the spiral.  $n = 19$  top left is about 19/8 or 2.3 turns and  $n = 43$  bottom left is 43/8 or 5.3 turns.

**Even  $n$ .** There are  $n-1$  first half jumps. But the  $n-1^{\text{st}}$  jump ends at  $n/2$  and includes both the curve emanating from vertex 0 and from vertex  $n/2$  as well as the transition diameter at jump  $n/2-1$  to  $n/2$  (jump 3 to jump 4 in the second half of the table given  $n = 8$ ). The spirals are symmetric, so we need only consider from 0 to  $n/2-1$  and double the resulting answer to account for the spiral starting at the bottom. The sum of these first  $n/2-1$  jumps is  $((n/2-1)+1) \cdot (n/2-1)/2 = n(n/2-1)/4$ , which, when divided by  $n$  and multiplied but 2 is  $(n/2-1)/2$  or  $n/4-1/2$ .

*Examples:* When  $n = 8$ ,  $n(n/2-1)/4 = 8 \cdot 3/4 = 6$ , just like the Jump 3 Sum in the bottom half of the table. This is 1.5 turns and resembles a [Yin and Yang](#) symbol. Top right  $n = 20$  is 4.5 turns, and bottom right  $n = 44$  is 10.5 turns.

**Note:** It is a bit difficult to count turns due to the sharpness of curves at the center of each image where the curves create scallops on each side of the halfway line rather than curves that rotate all of the way around. Also, for even  $n$  the transitional diameter (transitioning from the spiral starting at 0 to the one starting at  $n/2$ ) discussed earlier provides additional complexity. If  $n$  is divisible by 4, that diameter is horizontal, if  $n$  is divisible by 2 but not by 4, it is [vertical](#).

Below are 798 line  $n = 43$  and 1634 line  $n = 44$  spirals. Here is another  $n = 44$  image: [Yin and Yang with 2 Pentagrams](#).

