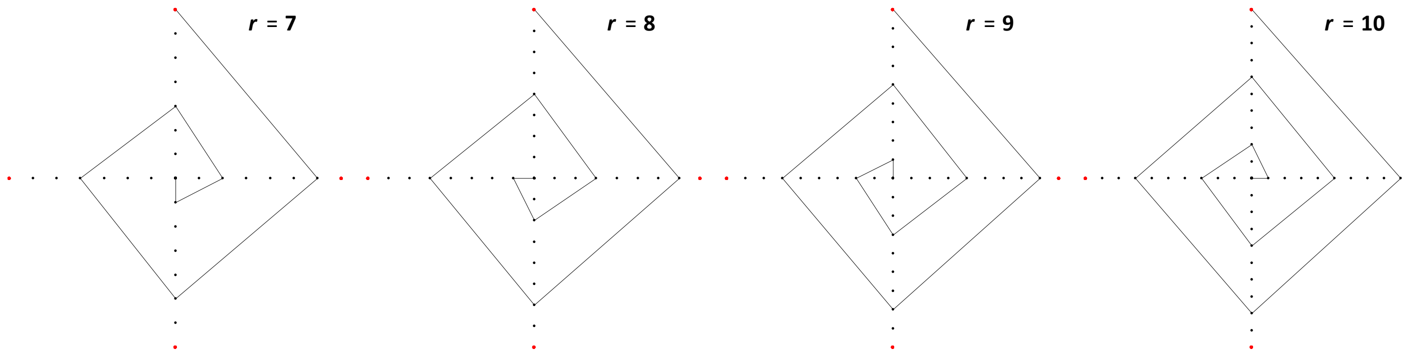
Almost Polygonal Sides are not Parallel: Some Square Examples

The images below show spirals created from ***n*** = 4 and ***J*** = 1 for 4 values of ***r***. The tables beneath each image show coordinates of the endpoint of each line as well as slope and quadrant associated with each line in the spiral.



r = 7				
line	***x***	***y***	Slope of line	Quadrant
0	0	1		
1	6/7	0	-1 1/6	I
2	0	- 5/7	5/6	IV
3	- 4/7	0	-1 1/4	III
4	0	3/7	3/4	II
5	2/7	0	-1 1/2	I
6	0	- 1/7	1/2	IV
7	0	0	Vertical	III

r = 8				
line	***x***	***y***	Slope of line	Quadrant
0	0	1		
1	7/8	0	-1 1/7	I
2	0	- 3/4	6/7	IV
3	- 5/8	0	-1 1/5	III
4	0	1/2	4/5	II
5	3/8	0	-1 1/3	I
6	0	- 1/4	2/3	IV
7	- 1/8	0	-2	III
8	0	0	0	II

r = 9				
line	***x***	***y***	Slope of line	Quadrant
0	0	1		
1	8/9	0	-1 1/8	I
2	0	- 7/9	7/8	IV
3	- 2/3	0	-1 1/6	III
4	0	5/9	5/6	II
5	4/9	0	-1 1/4	I
6	0	- 1/3	3/4	IV
7	- 2/9	0	-1 1/2	III
8	0	1/9	1/2	II
9	0	0	Vertical	I

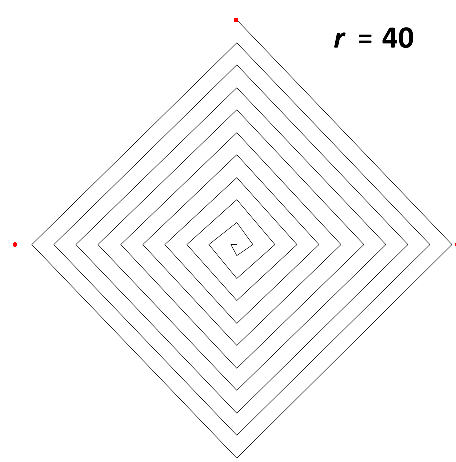
r = 10				
line	***x***	***y***	Slope of line	Quadrant
0	0	1		
1	9/10	0	-1 1/9	I
2	0	- 4/5	8/9	IV
3	- 7/10	0	-1 1/7	III
4	0	3/5	6/7	II
5	1/2	0	-1 1/5	I
6	0	- 2/5	4/5	IV
7	- 3/10	0	-1 1/3	III
8	0	1/5	2/3	II
9	1/10	0	-2	I
10	0	0	0	IV

In each image, the odd-numbered lines are in the first and third quadrants and even-numbered lines are in the 2nd and 4th. Notice that this pattern occurs because the spiral is drawn clockwise, starting in the first quadrant. Had we set ***J*** = 3, the first line would be in quadrant II with successive lines in III, IV, then I before starting around the second time.

The slope of odd-numbered lines increases in magnitude from |-***r***/(***r***-1)| > 1 for the first line. The steepest negatively sloped line (excluding the vertical line when ***r*** is odd) is the inner-most and has slope -3/2 when ***r*** is odd and -2 when ***r*** is even. The ***r*** = 40 image at the bottom maintains this pattern: the inner-most negatively sloped line has slope of -2.

The opposite pattern emerges in the 2nd and 4th quadrants. The steepest positively-sloped line is always the second line (outermost in quadrant IV) which has a slope of (***r***-2)/(***r***-1) < 1. Subsequent positively-sloped lines become flatter and flatter as one moves toward the center of the spiral. The flattest positively-sloped line has slope 1/2 when ***r*** is odd and 2/3 when ***r*** is even.

If you work outward from the center, you can readily see that even though the x and y coordinates and quadrant locations change, the slopes of lines depend on how far from the center is the line in question. For example, if you look from line 7 to 1 for ***r*** = 7, the slopes are the same as from line 9 to 3 given ***r*** = 9. The same pattern exists for spirals with even values of r: the slopes of line 8 to 1 for ***r*** = 8 match 10 to 3 for ***r*** = 10.



One final way to view this from the inside is to note that there are only 4 locations for the horizontal and vertical “last lines.” Each spiral ends at the center and each must either have started at 1/***r*** from the center along either the positive or negative x or y axis. The four alternatives are determined by the remainder once ***r*** is divided by 4. This means that the last 8 lines in the ***r*** = 40 image have the same slope **AND** quadrant location as the eight ***r*** = 8 lines above since both values of ***r*** are divisible by 4.