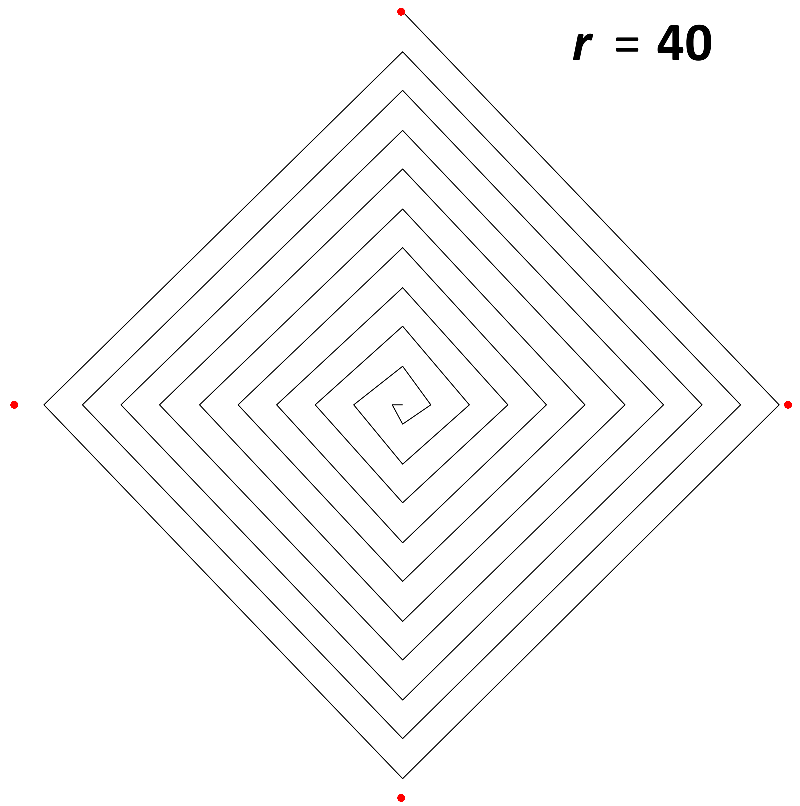
None of the Almost Square Angles are Right Angles



We know that seemingly parallel sides from almost polygons are not parallel to one another. Here we show that the angles formed by successive lines in almost squares are not right angles. **To form a right angle, the slopes of the sides forming the angle must be negative inverses of one another.** (There are other ways to show this, but they require knowledge of additional mathematical concepts.)

To simplify the discussion, assume that the almost square is not based on a unit circle but rather a circle of radius r. That way we do not need to worry about fractions and can simply discuss the angle created at the k^{th} endpoint.

Given $r = 40$, the 1^{st} endpoint is (39, 0) on the positive x-axis in the image at right. The angle created at this endpoint goes from the point (0, 40) to (39, 0) to (0, -38).

The 1^{st} segment slope is $\Delta y/\Delta x_1 = -40/39$ and the 2^{nd} segment slope is $\Delta y/\Delta x_2 = 38/39$.

The 1^{st} angle is not a right angle because the product of slopes is $-(40\cdot38)/39^2 = -1520/1521 \neq -1$.

The second angle (at (0, -38)) connects the second to 3^{rd} endpoint (-37, 0). The 3^{rd} segment slope is $\Delta y/\Delta x_3 = -38/37$.

The product of 2^{nd} and 3^{rd} slopes is $-38^2/(39\cdot37) = -1444/1443 \neq -1$, therefore the 2^{nd} angle is not right.

These are examples of angles given $r = 40$ for $k = 1$ and 2. For general r and k, the angle must have its vertex on either the positive or negative x-axis or y axis. The vertex location depends on the remainder when k is divided by 4.

+x) If k is a vertex on the +x-axis (i.e., $k = 1 \bmod 4$), the points are $(0, r-k+1)$ to $(r-k, 0)$ to $(0, -(r-k-1))$.

The slope of the k^{th} segment is $\Delta y/\Delta x_k = -(r-k+1)/(r-k)$ and the slope of the $k+1^{st}$ segment is $\Delta y/\Delta x_{k+1} = (r-k-1)/(r-k)$.

These slopes are not at right angles because the product of slopes $= -[(r-k+1)(r-k-1)]/(r-k)^2 = -[(r-k)^2 - 1]/(r-k)^2 \neq -1$.

-x) If k is a vertex on the negative x-axis (i.e., $k = 3 \bmod 4$), the points are $(0, -(r-k+1))$ to $(-(r-k), 0)$ to $(0, r-k-1)$.

The slopes and product of slopes are the same as above for the positive x-axis case, so the k^{th} angle is not right.

-y) If k is a vertex on the negative y-axis (i.e., $k = 2 \bmod 4$), the points are $(r-k+1, 0)$ to $(0, -(r-k))$ to $(-(r-k-1), 0)$.

The slope of the k^{th} segment is $\Delta y/\Delta x_k = (r-k)/(r-k+1)$ and the slope of the $k+1^{st}$ segment is $\Delta y/\Delta x_{k+1} = -(r-k)/(r-k-1)$.

These slopes are not at right angles because the product of slopes $= -(r-k)^2/[(r-k+1)(r-k-1)] = -(r-k)^2/[(r-k)^2 - 1] \neq -1$.

+y) If k is a vertex on the positive y-axis (i.e., $k = 0 \bmod 4$), the points are $(-(r-k+1), 0)$ to $(0, r-k)$ to $(r-k-1, 0)$.

The slopes and product of slopes are the same as above for the negative y-axis case, so the k^{th} angle is not right.

Each product uses the *Difference between Squares* formula: $a^2 - b^2 = (a-b)(a+b)$ (with b = 1), discussed elsewhere.

Using the $r = 40$, $n = 4$ and $J = 1$ image above, the vertices on the positively-sloped part of the y-axis each have x = 0 but have y = 36, 32, 28, 24, 20, 16, 12, 8, and 4, as k ranges from 4 to 36 by 4. By contrast, the vertices on the positively-sloped part of the x-axis each have y = 0 but have x = 39, 35, 31, 27, 23, 19, 15, 11, 7, and 3 as k ranges from 1 to 37 by 4.