Aestheometry: Creating Art while Learning Math using Electronic String
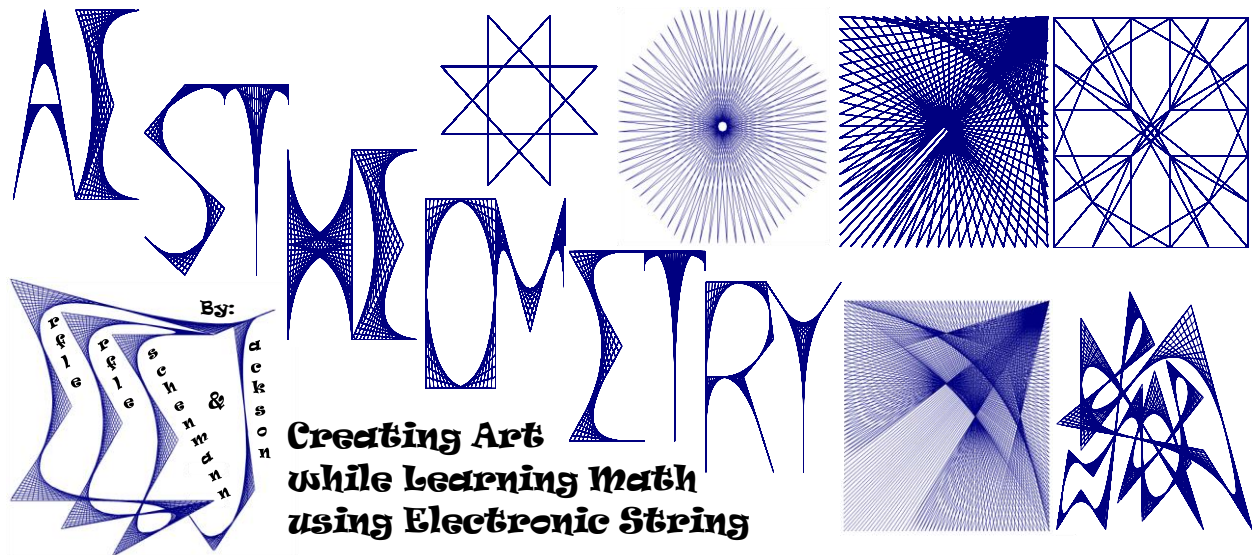
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December 9, 2019

Based on the Excel file: Aestheometry_EEEJ.xlsx

Aestheometry: Creating Art while Learning Math using Electronic String

Abstract

The Aestheometry.xlsx file, originally created to encourage summer-time explorations for anyone who is familiar with Cartesian coordinates, can be used for exactly that purpose or as a classroom activity for students learning Cartesian coordinates. This file mimics, in an electronic fashion, string art. Sometimes string art projects start from curved shapes but even when the underlying image (i.e., where the string is attached) is piecewise linear, curves result. Aestheometry is the term for creating patterns and curved images using straight lines. This paper discusses the implementation of this file in a classroom and provides a guide to the file. Students learn the difference between the three parameters that essentially run the file, vertices, subdivisions, and points, via an initial series of annotated worksheets. Subsequent worksheets allow students to change the shape of the image by changing vertex locations. The main sheet allows students to create their own images with up to 21 vertices. Students can create images which can only be described as artistic. The goal is to provide an enjoyable educational experience where students want to play with the file and create their own images. As they do this, they learn math in a creative way.

This paper serves multiple purposes. First and foremost, this paper provides a guide to the Aestheometry.xlsx file (hereafter the file). This guide, which starts in Section 1, provides instructors with enough familiarity with the file that they can teach from it in a classroom setting. The last page of the guide (and the Handout sheet in the file) is an in-class handout. Interested students could be provided with the full guide because it is written in a non-technical fashion, but it is not necessary for students to read the full guide, especially if they have been introduced to the file in a classroom setting. The file is extensively annotated, especially in early sheets (which

are numbered and lettered for organizational purposes), because the file was created for independent student usage, outside of the classroom. Before turning to the guide, we discuss **A)** the mathematical concepts that can be highlighted using this file, **B)** how this material can be presented in the classroom, and **C)** the results of a questionnaire that examined students' in-class experience.

A. Mathematical Concepts) The file is based on graphing line segments between points in (x, y) space. **The file helps students gain familiarity with the Cartesian coordinate system**. Students could benefit from early sheets in the file (2 – 4c) even if they do not know how to think about points in (x, y) space, but the file was created to help students solidify their understanding of this notion in a fun and engaging fashion.

A second mathematical concept encountered here is **how to represent three-dimensional objects as two-dimensional images** in sheets 4a, 4b, 5a, and 5b. Those just learning (x, y) coordinates could certainly benefit from playing with the x or y coordinate of the top of the pyramid and watch what happens to the image of the pyramid in sheet 5a.

Secondarily, the file could be used to examine **fractions** because that is how subdivisions between vertices are created. Students need not follow the hyperlink on sheet 3a (which takes the user to a discussion of how these subdivisions are created on sheet 7 starting in cell A48) to use the file. But instructors wanting to teach fractions in an interactive fashion, could use this part of the file for this purpose. Similarly, one need not focus on the concept of **greatest common divisor**, GCD, to play with the file, but it turns out that the number of distinct connected line segments used to create an image, N, is given by the equation N = (V·S)/GCD(V·S, P). This is initially discussed in sheet 2 at cells with light green highlighting starting at cell U19 but this same concept

appears later in the file as well (when complex images collapse into very simple ones as P changes for fixed V and S). This is why the **primes** < 100 is listed in cells D3:D27 on all sheets (except 2).

B. Classroom Implementation) This file has been used in a variety of middle-school and high school classrooms. Prior to class, the instructor loaded the file onto classroom management software to provide students access to the file. Students were instructed to download the file and make sure they had their laptops or iPads fully charged.

Sheets are labeled with descriptive names in addition to the numerical and letter coding that describes the hierarchy of sheets in the file. Sheet 2 only allows P to vary. Sheets 3a and 3b allow S and P to vary, and sheets 4a – 4c allow all three parameters to vary. Sheets 5a and 5b show how images change as individual linked vertices change and sheet 6 allows full access to vertices as well as V, S, and P. Table 1 provides an outline for instructors wanting to introduce the file in a classroom. Values of V, S, and P as well as the sheet used are noted in Table 1.

The instructor started class by passing out the Handout and asked students to open the file to the Title page which shows images created by the file. (Make sure to print this single-page handout in color as it is color-coded to highlight the distinction between the three parameters that control this file, V, S, and P.) Students were encouraged to resize their page due to the size of their screen. The instructor quickly showed the 1.BasicGraphing sheet, just so students knew how to draw a graph in Excel before moving to the first substantive sheet, 2.Square, which introduces the parameter P to students. Instructors will end up spending more time discussing the first few images than later ones as students have to become familiar with how the parameters control the resulting image. As a result, the discussion below will focus on those first few images.

Table 1

Instructor Notes for a 50 minute Introduction to Aestheometry Class*

V	S	P	Sheet	Talking points for resulting image with Q&A suggestions
A1	B1	C1	1	This sheet shows how graphs are made in Excel.
4	5	4	2	The square has V = 4 vertices with S = 5 subdivisions between vertices.
				Since P = 4, the resulting pentagon includes only the first vertex.
		8		This star was created exactly how students typically draw stars.
		9		Tight-center 20-point star. P = 11 is the same image with order reversed.
4	2	3	3a.	This is the annotated image on the Handout showing V, S, P.
		4		The image collapses because P is a common divisor of V·S.
		3.5		Now, segments alternate between P = 3 and P = 4.
	23	11		Each curve does not extend halfway since P/S < 0.5, compare P = 21 or 45.
8	23	11	3b.	The image appears closer to a circle than on sheet 3a. Try P = 45 or 91.
20	43	23	4a.	The opening image is the cube. To see how it was built, change V.
4				Now the front is the only thing you see. Set V = 3 and see a triangle.
5				The top-right corner has been added … and a strange shape results.
8				Now the front and top are visible.
12				Now the three visible sides are visible.
16				Note that the last USED vertex is (4, 14). From here, go back to (10, 0).
20	43	23	4b.	This is simply a different view of a cube but it works just like 4a.
5	23	11	4c.	Change V from 5 to 20 to see the Handout arrow piercing the heart image.
20	37	17	5a.	Initial peak value is X = 0.6, Y = 6 in V7:W7. Note: In this instance, the left side of pyramid is in the "line of sight." Q: How would you make a shorter pyramid by changing X or Y? (A: Set Y < 6 in W7.)
Y = 3 in W7				Note that the left side is visible when Y < 6. Q: How would you make a pyramid in which the top is not "directly over" the base? (A: In V7, tilt the top to the left by setting X < 0 or to the right by setting X > 1.)
X = -0.6 in V7				Check what happens if X = -10. Now the base looks like it is the side! Return X = 0.6 and ask Q: How could you show what it looks like if you fly above the pyramid? (A: 0 < Y < 1 in W7.)
Y = -3 in W7				Compare with Y = 3, note that the front and back faces appear to switch.
17	41	13	5b.	The 3-D nature of this pyramid depends on how large P is relative to S. Change P to 23 and see a 4-leaf clover instead of a pyramid.
		13		Reset P = 13. Q: Can you adjust any of the vertices to make the image more three-dimensional? (Ask students what they did.)
21	43	23	6	Now students can create their own images by setting vertices in B6:C26 as well as V, S, and P. Q: Given this "mountain" image, what would you do to make the middle peak higher? (A: Set cell C24 > 9.)
5	53	29	7	The curve at (1, 1) is captured by repeating this vertex twice in a row.

*Each sheet is controlled by 1 to 3 cells, A1, B1, and C1, that are highlighted in yellow. Other cells are protected.

As noted above, P is the only parameter to vary on sheet 2, the other two parameters are fixed (at V = 4 and S = 5 meaning there are five subdivisions along each of the four sides of the unit square). The initial image shows a pentagon because there are four subdivisions between successive points, P = 4. Line segments are drawn between points with the first segment starting at the first vertex, (0, 0), and ending after 4 subdivisions at (0, 0.8). The next point is 4 subdivisions away at (0.6, 1) and the next two are 4 subdivisions away at (1, 0.6) and (0.8, 0). The final point brings us back to the initial vertex (since there are 4 subdivisions between (0.8, 0) and (0, 0)). If instead, you wanted to create a star, set P = 8. The resulting image is exactly how students make stars on a piece of paper: start at one of the corners of the pentagon (here (0, 0)) and then jump past the next corner to the one two corners away. Do this jump four more times and you have traced out a star. From here, the instructor suggested: Try other values of P between 1 and 10 and see what happens (the bottom left and center images on the Handout are two examples). The images that result are discussed in notes to the right of the figure in sheet 2.Square.

Sheet 3a.Square allows students to vary S as well as P. The large 8-point star image in the Handout occurs when S = 2 and P = 3 (given V = 4). **It is worth explicitly telling students to look at the Handout as it annotates what each parameter means in this context.** From here, change P to 5 and see what happens. It appears like nothing happens but in actuality, the first point is now more than half way around the square at (1, 0.5) rather than (0.5, 1). The image collapses to a line segment when P = 4 because the greatest common divisor of V·S = 8 and 4 is 4. An interesting thing occurs if P = 3.5. In this instance, jumps alternate between 3 and 4. A number of other possibilities are suggested to the right of the image, but it suffices to show one or two of them to provide enough information to move on. S = 23 and P = 11 provide a nice graphic image because

the corners look very much like curves, despite the fact that each curve is made from 10 line segments. Have students try the same values for S and P on sheet 3b.

The opening image on sheet 4a is a cube based on V = 20. Change V as noted in Table 1 and see how the cube was built. Sheet 4b provides a different perspective on the cube and 4c provides a similar introduction to the parameter V in the context of a more abstract image.

One could spend a lot of time playing with sheets 5a and 5b but that need not happen. These sheets are provided so that students can see how to create a figure that they can then readily move by linking vertices to a base set of (x, y) coordinates listed elsewhere on the sheet. Follow the instructions in Table 1 to see some of the possibilities. The various scenarios for sheet 5a in Table 1 are presented with Q&A format so that the instructor can prompt students to consider how x and y movements of the coordinates of the peak vertex affect the shape of the rectangular pyramid.

Sheet 6.Aestheometry provides students access to all vertices as well as V, S, and P. Little time needs to be devoted to this sheet in class. Simply point out that students can change individual vertices given the image provided or they can delete all vertices and create their own image from scratch. When creating their own image, it helps to start with S = P = 1 so that only vertices are shown. Once the outline is created, then vary S and P to create curves on the superstructure they have created. Students can also copy and paste vertex values from sheet 8.SomeShapes onto sheet 6 then adjust V, S, and P as suggested and see what images result. Sheet 7 shows how to incorporate asymmetries into the image by listing a vertex twice in a row. This effectively swallows up the curve at that vertex unless P > S in which case the curve at that vertex

is smaller than at other vertices. Instructors can ask students: Which of the images on the Title sheet were created using this trick?

C. Survey Results) Upon completion of the lesson, students were given a questionnaire. The results of the questionnaire are provided in Table 2. These results suggest that students enjoyed the file but do not want to have homework based on the file. On the other hand, they would like to have extra credit given based on the file. In general, students thought the Handout was well-laid out as was the file itself. Sixty-five percent thought the file would be helpful in teaching younger students (x, y) coordinate and more than forty percent thought the file could be helpful in teaching fractions and the notion of greatest common divisor.

When students were asked which grade level they thought would be appropriate to introduce these concepts/activities, over half felt that the 5th or 6th grade was the right time to introduce the file. This is heartening, as the file was originally created as something students could play with over the summer between their 5th and 6th grade years.

Table 2
Results of Questionnaire regarding Classroom Implementation of the Aestheometry Excel file*

92.8	1. Did you enjoy playing with the file today?
84.3	2. Did the Handout offer sufficient explanation of how the file works?
	A. If no, can you suggest improvements?
22.3	3. Would you be interested in being given homework based on this file (such as "Create an image showing your initials" or "What is the easiest way to make a figure 8?") using this file?
68.9	4. Would you do extra credit problems based on this file if that were offered as an option in this class?
87.3	5. Did you find the progression of worksheets teaching P, S, and V helpful?
	B. If no, do you think this should have been done with fewer sheets prior to 6.Aestheometry?
89.3	6. Do you have access to a computer to use at home that has Excel on it?
65.1	7. Could you envision using this to help teach (x, y) coordinates to younger students?
42.7	8. Could you envision using this to help teach fractions or the notion of greatest common divisor to younger students?
	C. Think about earlier in your math career. What is the earliest grade that you think you might have benefited from playing with this file?_____
41.7	9. Do you have younger sisters or brothers who might enjoy playing with this file?
28.4	10. Would you like to have a copy of the 3-color extension to the file you played with?
	D. Finally, I would like to give you the chance to make suggestions for improvements that could be made. For example: Would you like more "build out" on any sheet in particular? or, Is there a specific part of the file you are still confused by?

*Percentage yes responses from 84 students (43 from three high school geometry classes, 22 from one 8th grade Algebra II class, and 19 from mixed grade after school classes) for all yes/no questions (response to individual questions ranged from 81 for Q10 to 84 for Q6 and Q9). One student added a free response of "I do not like HW" to Q3 and a number of students noted "No younger siblings" in response to Q9. The histogram below shows the cumulative results of 58 students who provided answers to Question C.

Earliest Grade to Introduce this File
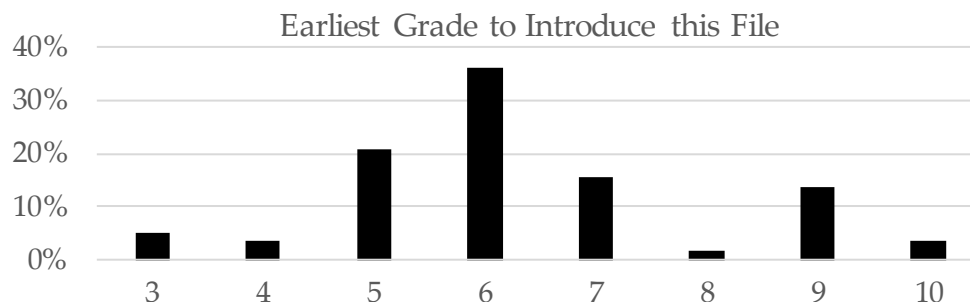
1. Introduction to the Guide

This guide accompanies an Excel file that allows students to create artistic patterns by playing with points on a graph. It uses the idea that one can create images that appear to curve using straight lines. The term for creating curved images using straight lines is *aestheometry* and it is often done as "string art." This file helps students strengthen their understanding of the Cartesian coordinate system in an active-learning and fun way. As they do this, they will learn more about prime and relatively prime numbers as well as how to model 3-dimensional figures within a 2-dimensional system.

The model is built up over the course of a series of Excel sheets. The full model, shown in sheet 6.Aestheometry, allows the user to change vertex locations, as well as the three parameters, V, S, and P, that run the graphics displayed (V, S, and P will be defined shortly). Prior sheets show how these three parameters work to create various images. Sheets begin with a number and sometimes an a, b, or c to provide some hierarchy between sheets. Instructions are provided on each sheet to help you understand that sheet and these instructions become less detailed as we move to higher numbers. The final page provides a handout you can use as you work through the file (and that can be used as a classroom handout).

Start with the sheet 1.BasicGraphing. This is provided so you know how to create a basic graph in Excel. Follow the instructions for graphing a unit square. Although you will not have to do this for the rest of the file, this is provided so you can see how all the other graphs in the file were created.

2. Exploring the number of subdivisions between points, P

Sheet 2.Square is where we really begin. This sheet focuses on parameter **P**, the number of subdivisions between points, highlighted in yellow and located in cell C1. This is the only parameter you can change on sheet 2.Square. In order to understand P, you need to understand a bit about S and V (even though you cannot change these parameters on this sheet). The definitions of all three are provided beneath the graph in cells D30:M44 (in Excel, we talk about a block of cells using a colon between the upper left-most, here D30, and the lower right-most, here M44). Please read these definitions before following the instructions on that sheet. We expect that you will not completely comprehend these definitions prior to playing with all three of the parameters in later sheets (sheets starting with 3 allow you to vary S and P and sheets starting with 4 introduce V).

The vertices of a unit square, (0, 0), (0, 1), (1, 1), and (1, 0), are shown in B6:C9. The sheet is programmed to use these vertices **in the order that they are listed** and to create points to be connected by line segments used to create each image. The number of vertices used, V = 4, and the number of subdivisions between vertices, S = 5 are fixed on sheet 2. You only can change the number of subdivisions between points, P. Start by following the instructions to the right of the sheet and try P values from 1 to 10 in C1. Figure 1 shows a screen capture of one example, P = 3 (we have added further detail (six short red arrows, six long blue arrows, and some line segment numbering) to what you see on the Excel sheet). In Figure 1, each red arrow represents a subdivision. Since S = 5, there are 5 subdivisions between (0, 0) and (0, 1). The blue arrows, tracing out the shape the figure makes, occur every 3 red arrows (since P = 3). The first line segment, labelled 1 at the pointed end of the vertical blue arrow, goes from Point 0, (0, 0), to Point 1, (0, 0.6)

(see figure and coordinates in R3:S4). The figure is based on 20 line segments: the first six of which are shown as numbered blue arrows and the remaining diagonal segments are labelled by line segment number. Notice that the figure is traced out by going clockwise around the square three times before reaching the starting point (0, 0) once again after the 20th line segment (from (0, 0.6) to (0, 0)).

Points in each figure are laid out using vertices and subdivisions between vertices. The coordinates of these points are in columns R and S to the right of the graph in Figure 1. Once again, P is the number of subdivisions between successive points and the figure changes as P changes. Each figure is made up by connecting line segments until the pattern repeats. The pattern repeats after V·S points, here 20, as long as P has no common divisor with V or S. When P has a common divisor, the pattern repeats in fewer than V·S points. (You can see the general equation for pattern repetition in U46, but that is not important for our purposes.)

When P has a common divisor with V or S, some vertices may be excluded from the final graph. Figure 2, with P = 2, provides one such example. All ten line segments are shown with numbered blue arrows and each line segment contains P = 2 subdivisions (note the small red subdivision arrows from Figure 1 are suppressed in Figure 2). In this instance, the second and fourth vertices, (0, 1) and (1, 0), are excluded from the final graph by the 3rd and 8th segments. For some values of P only the initial vertex is included in the final image, as when a pentagon is formed given P = 4. In this instance, only the initial vertex, (0, 0), is included in the resulting image.
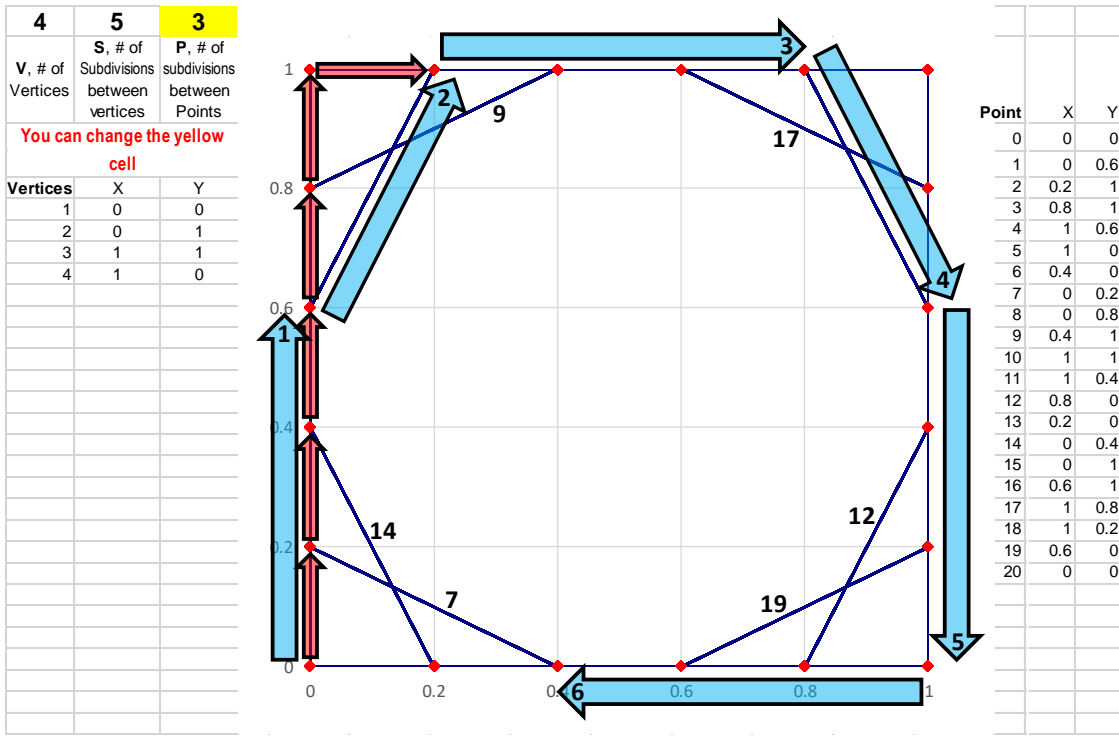
4	5	3
V, # of Vertices	**S**, # of Subdivisions between vertices	**P**, # of subdivisions between Points

You can change the yellow cell

Vertices	X	Y
1	0	0
2	0	1
3	1	1
4	1	0

Point	X	Y
0	0	0
1	0	0.6
2	0.2	1
3	0.8	1
4	1	0.6
5	1	0
6	0.4	0
7	0	0.2
8	0	0.8
9	0.4	1
10	1	1
11	1	0.4
12	0.8	0
13	0.2	0
14	0	0.4
15	0	1
16	0.6	1
17	1	0.8
18	1	0.2
19	0.6	0
20	0	0

Figure 1: Screenshot of sheet 2.Square cells A1:S30 showing four vertices of a square in B6:C9 and the 20 line segments in the graph based on points 0 – 20 in R3:S23, given V = 4, S = 5, and P = 3.
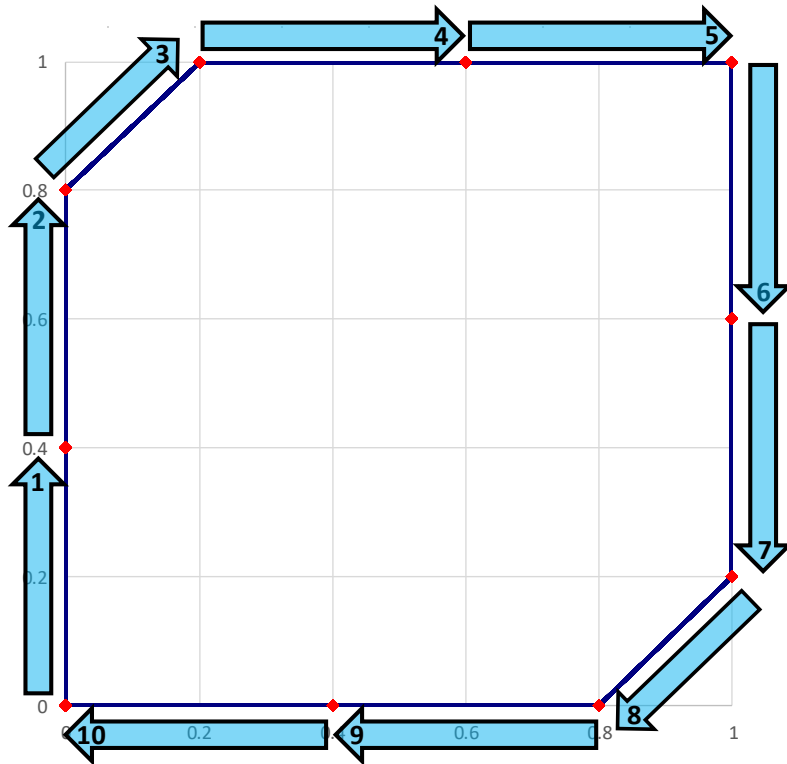
Figure 2: Sheet 2.Square with S = 5 and P = 2 shows that the 2nd and 4th vertex will never be points in the final image because there are 2 subdivisions between points and there are 5 subdivisions between vertices.

3. Exploring the number of subdivisions between vertices, S

Sheet 3a.Square uses the same vertices as 2.Square, but now you are able to change the number of subdivisions on each side, **S**, as well as P. This means you can change how many "red arrows" would fit between vertices on Figure 1. When S = 1 and P = 1, vertices and points coincide because there is only 1 subdivision between each point and only 1 subdivision between adjacent vertices (the whole side). The next suggested value, S = 2 and P = 3, gives us the shape shown in Figure 3 (and shown as the main figure in the Handout). Since P = 3, the first line segment drawn from Point 0 to Point 1 has to skip 3 subdivisions, and because S = 2, there are 2 subdivisions between each vertex. The final figure, an 8-pointed star, is mapped out by completing 3 clockwise trips around the square.
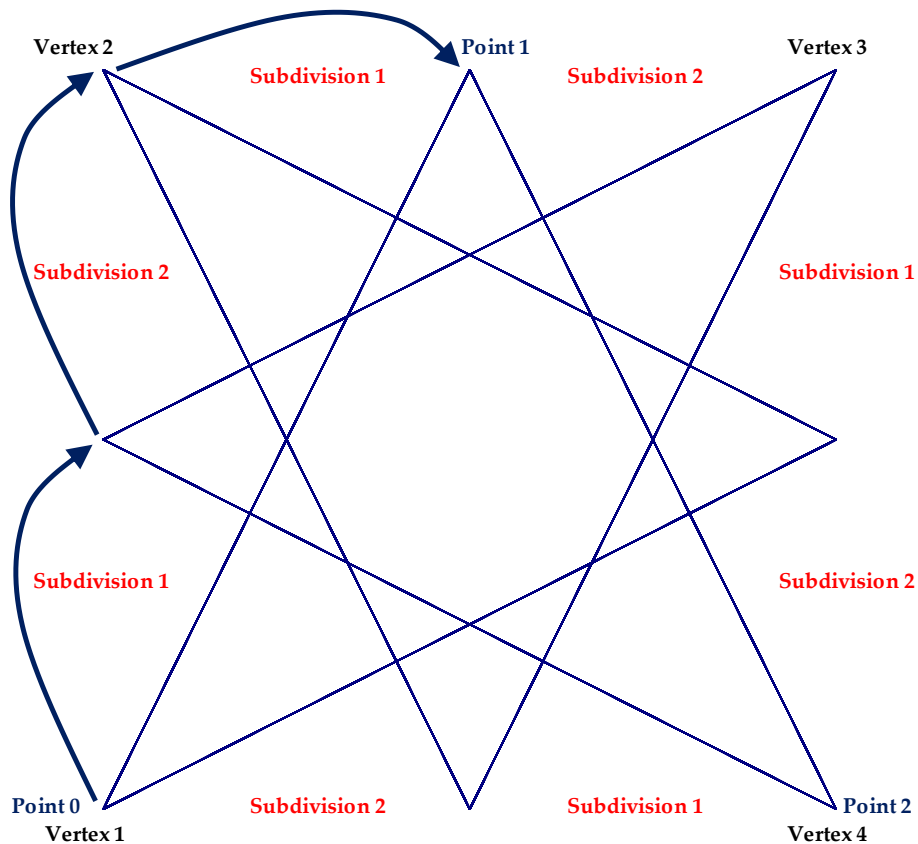

Figure 3: Sheet 3a.Square with S = 2 and P = 3 showing jumps required to create the 1st segment of the 8-pointed star.

If we trace this pattern all the way around until we arrive at our starting point, we are left with an 8-pointed star. If you continue to number the points along the outline of the square, you will see the full pattern followed to form the star. The result is Figure 4A. If we set $P = 5$, we obtain Figure 4B. The shape looks exactly the same as 4A! However, if we trace it out like we did in Figure 3, we see that the order appears reversed; the first line is drawn after the 5th subdivision instead of the 3rd. One might view this as creating the star in a counterclockwise direction, but in actuality we are simply moving more than half way around the square before the next point is placed (we move 5/8 of the way around the square in Panel 4B but only 3/8 of the way around the square in Panel 4A).

In general, when $P > S$, star shaped figures result. Star-shaped figures occur whenever sequential points are **necessarily** located on different vertices, a condition that is guaranteed to happen only if $P > S$. (But $P > S$ does not guarantee a star will result. Set $S = 5$ and $P = 23$. You should try to think through why the resulting is the same as in Figure 1.)

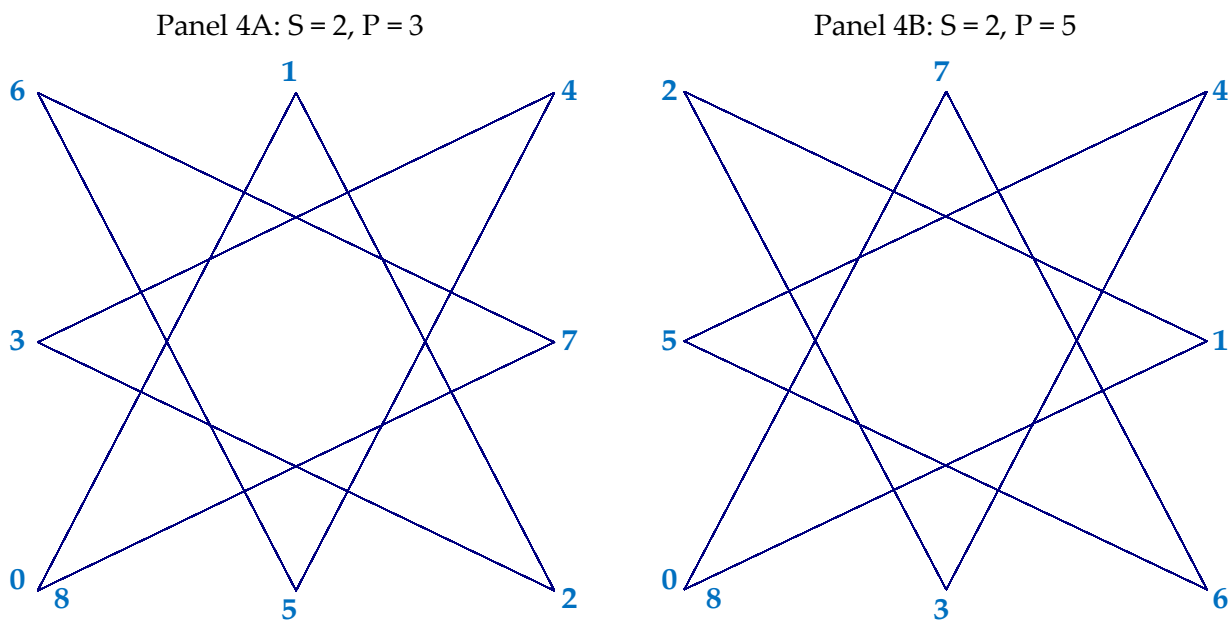Panel 4A: S = 2, P = 3   Panel 4B: S = 2, P = 5

Figure 4: Figure 3's pattern can be obtained in multiple ways. Points are labelled in the order they are plotted.

Continue on the 3a.Square sheet by plugging in the S and P values suggested and compare your own thoughts to the comments on the spreadsheet. Figure 5 begins to show how Cartesian coordinate points connected by line segments can create curve-like figures. The curves in each corner are formed by 10 line segments (since P = 11), creating the illusion of a curved edge.
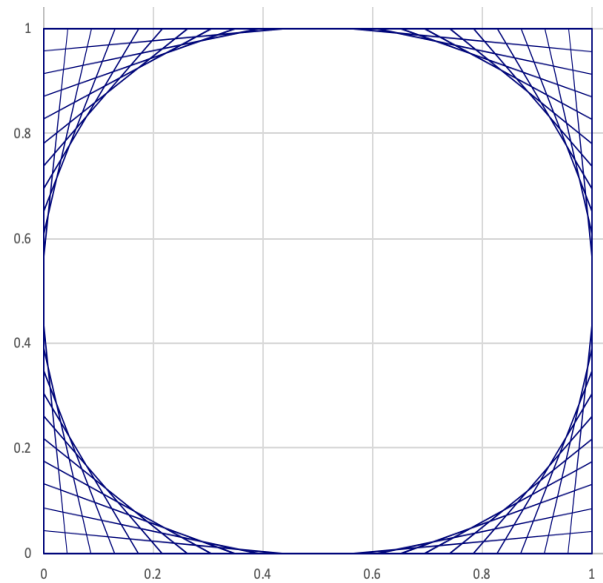

Figure 5: Sheet 3a.Square with S = 23 and P = 11, we are shown a figure that has a curve in each corner.

Thus far, we have only worked with vertices forming a square. Sheet 3b has 8 vertices in the shape of an octagon. Images change because vertices change as we see in Figure 6. It is worth examining some of the same S and P combinations we have already used with the square and see what happens with an octagon. Figure 6A shows the same S and P values as Figures 3 and 4A, and Figure 6B shows the same S and P values as Figure 5. In the first instance, because P > S we have a star. This star has 16 points (rather than 8) and in the second instance, we have smooth curves that now even more closely approximate an interior circle (based on the octagonal frame of vertices). The shapes look very different due to the difference in the location of the vertices.
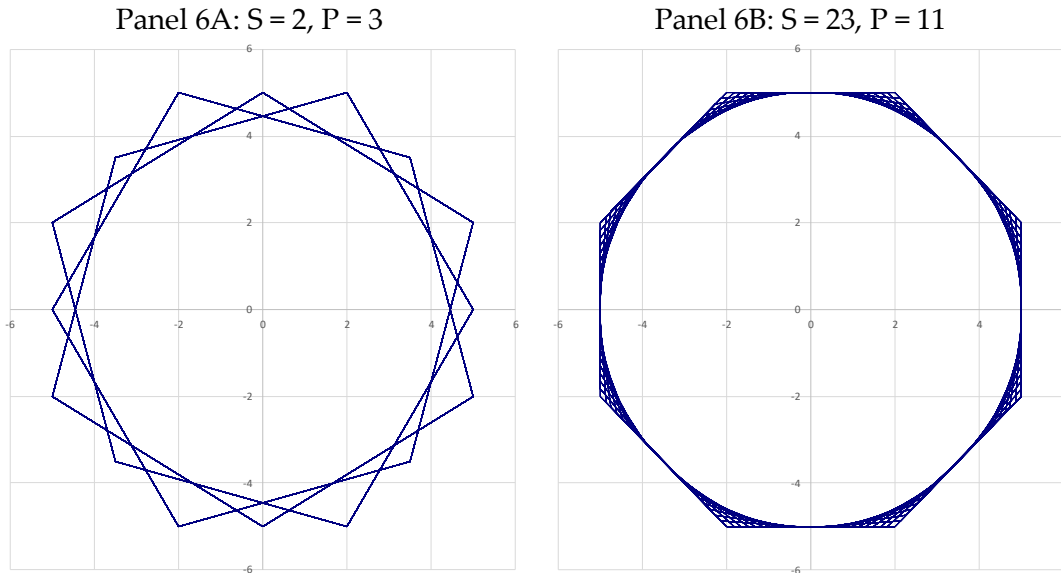
Panel 6A: S = 2, P = 3   Panel 6B: S = 23, P = 11

Figure 6: Sheet 3b.Octagon for various S and P values.

4. Exploring the number of vertices used, V

The third parameter, V, has to this point been held fixed (at V = 4 when we worked with the square, or 8 when we worked with the octagon). Up until now, all vertices shown are used to create the final graphic. In each instance, the first vertex is in cells B6:C6. If we made a distinction between the actual vertices starting in B6:C6 and those USED, then we could have different shapes from the same set of vertices by changing **V**, the number of vertices used. For example, if we only use the first 3 vertices of the square in 2.Square or 3a.Square we would end up with a triangle, in fact we would have a right triangle with the right angle at the point (0, 1). This could be accomplished by setting V = 3 in cell A1 (that cell is locked so you cannot do that on those sheets, but you should be able to think through what it looks like). We will now turn to sheet 4a.CubeA which will allow you to vary V as well as S and P.

The complete cube in sheet 4a.CubeA required 20 vertices. (Interestingly, a cube only has 8 vertices, but if you use only 8 the resulting image does not look like a cube. You will soon be

able to try this yourself in sheet 6.Aestheometry.) By changing the number of vertices used, V, we can see how the final image was created. The four panels shown in Figure 7 focus attention on how the cube was created as V varies. The sides of the cube could have been created in any order, but we chose to draw the front of the cube first, followed by the top, followed by the visible (right) side, then back and hidden (left) side.
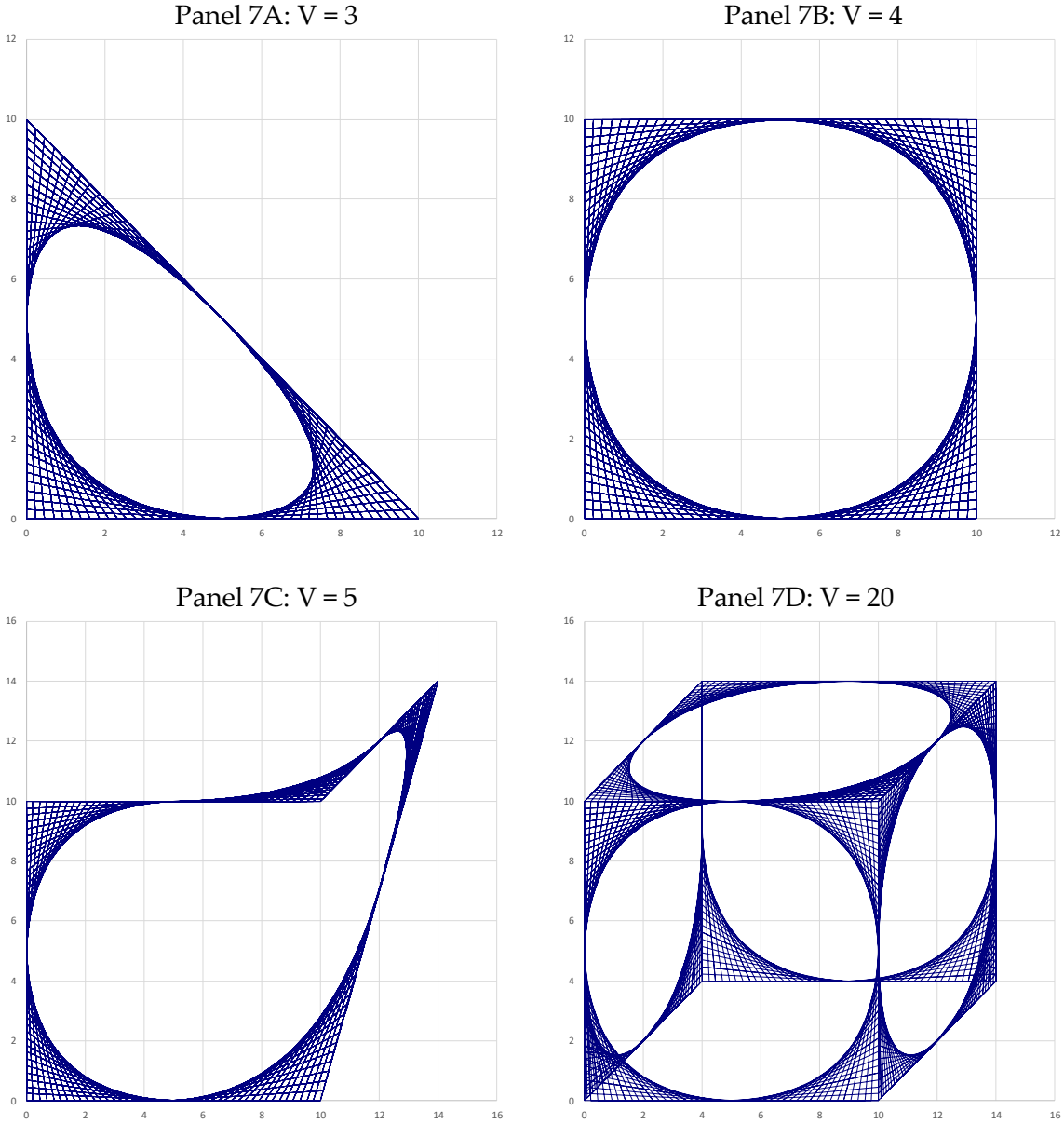
Panel 7A: V = 3

Panel 7B: V = 4

Panel 7C: V = 5

Panel 7D: V = 20

Figure 7: Sheet 4a.CubeA for various values of V, given S = 43 and P = 23.

The initial vertex, in the bottom-right-front corner, is (10, 0) and subsequent vertices of the front square are listed in clockwise rotation. Panel 7A shows the isosceles right triangle of vertices that result when only the first 3 vertices are used. The important point to note is that this figure did not specifically list (10, 0) as a fourth vertex. The program which creates the graphic image ALWAYS returns to the first vertex after the final **used** vertex is found. This was initially discussed in sheet 3a.Square cell X9, but it is worth reminding you now that you can change the number of vertices used, V. With that in mind, Panel 7B looks just like what you would have seen had you set S = 43 and P = 23 in sheet 3a.Square (except this square has sides that are 10 units long instead of one unit long). The fifth vertex at (14, 14) is the top-back-right corner of the cube. But, given V = 5 in Panel 7C, the resulting figure looks quite strange (and V = 6 and V = 7 look even stranger). When V = 8, the front and top of the cube are shown, and the right side becomes visible when V = 12. Once V = 20, the cube is completed as seen in Panel 7D.

Sheet 4b.CubeB gives us the same object shape as 4a.CubeA, but from a different perspective. This change in perspective is, of course, due to a different location of vertices. Glancing at the first vertex, we can see that on sheet 4b.CubeB, it is located at (3, 0), compared with 4a.CubeA at (10, 0).

The next sheet shows how unique shapes, such as an arrow piercing a heart, are formed. All we must do is make sure the vertices are placed in the right spots in the correct order. This has been done for you in 4c.Arrow&Heart. You can slowly watch the shape emerge by changing V from 5 to 20, increasing one number at a time. Each increase in V incorporates another vertex into the shape. As in earlier sheets, the pre-programmed vertices are in B6:C25. Alternatively, change V = 5 to V = 20 all at once to see the two images on the left side of the quick-start guide.

5. Moving linked vertices

We have seen how V, S, and P change a graphic image, but what happens if we change the location of the vertices themselves? On the next sheet, 5a.PyramidA, we do exactly that; change the location of a pyramid's peak vertex. The x and y coordinates of the peak, highlighted in yellow in cells V7:W7, are shown in Figure 8. Those two values are linked to the purple vertices (5, 8, 13, 16, and 19) listed in B6:C25. Changing the x or y coordinate can change the look of the shape dramatically without ever changing V, S, or P. Instructions on the sheet allow you to explore this topic. An especially interesting thing happens when you invert the pyramid by setting the y coordinate of the peak to -3, for example.
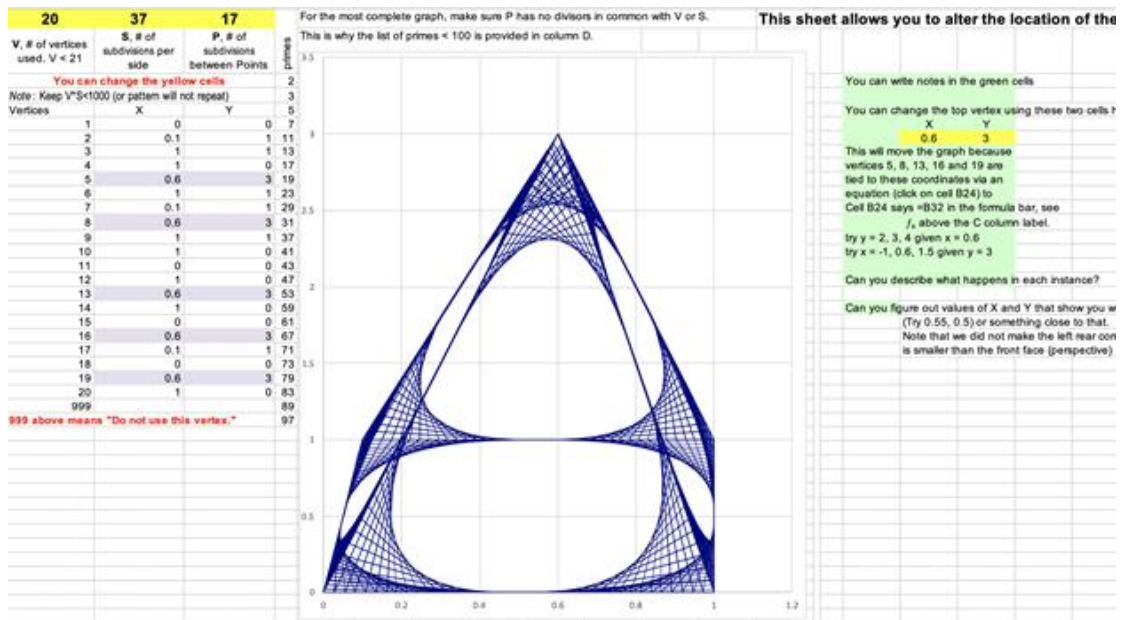

Figure 8: Change what the pyramid looks like by changing the x or y coordinate of the peak on sheet 5a.PyramidA.

Extending this idea, sheet 5b.PyramidB links all four vertices of a triangular pyramid in V5:W8 to the vertices in B6:C22. This provides even more shape flexibility, as each of the pyramid's vertices are linked with the "main" vertices to the left of the shape (just like what we saw in Figure 8 for the peak vertex).

6. Creating your own pictures

It is now time to put all the pieces together. The sheet 6.Aestheometry allows you to adjust vertices in B6:C26, highlighted in pink. See if you can form your initials, or a cool shape. When creating a new pattern, it sometimes helps to set S = 1 and P = 1 so you only see the vertices. Remember, there are 4 components to creating any image: the location of the vertices in B6:C26; V, the number of vertices included in the figure, in A1; S, the number of subdivisions between adjacent vertices (sides), in B1; and P, the number of subdivisions between points, in C1. Two versions are provided on sheet 6.Aestheometry, one with the coordinate system visible, and another that hides the coordinates so you can print images without reference to the underlying axes that you used to create the image. To learn more about printing the image, see the discussion starting in cell U29. Finally, 6.FullPage allows you to change the color of the image.
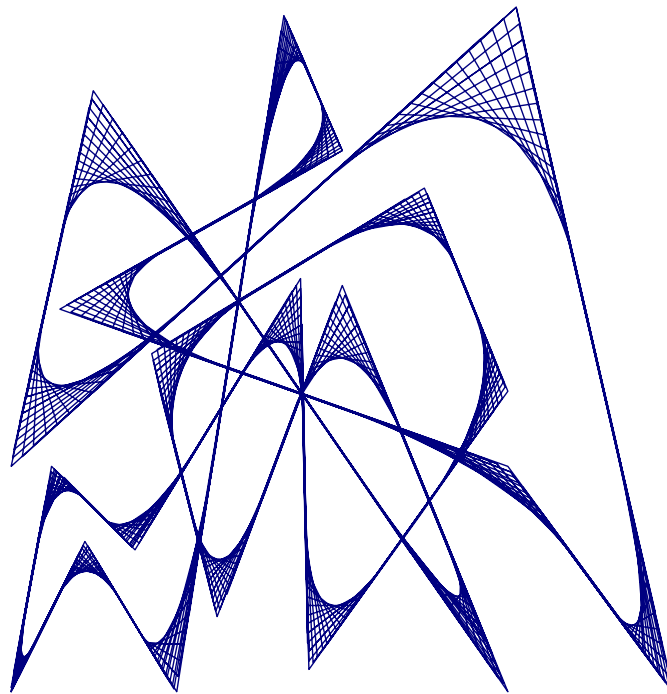

Figure 9: Image from sheet 6.Aestheometry with an abstract "mountain" scene.

Figure 10, based on sheet 7.A_trick, shows what happens if you repeat the same vertex. The square was created with 5 vertices not 4. The 3rd and 4th were both (1, 1). This allows you to "capture" the curve at that vertex. This capture occurs as long as P < S but once P > S we see a secondary curve coming out of the (1, 1) vertex. For example, if S = 5 and P = 9, some of the 20-pointed star that you saw when you set P = 9 in 2.Square is visible, but part has been captured by (1, 1). In addition, there is a secondary curve at (1, 1). If you cannot see the secondary curve, try a larger combination such as S = 53 and P = 84, the value suggested in the notes on this sheet at A37. Now that you have seen this trick, it should be clear which images on the Title page used this trick. See if you can recreate those images using 7.A_trick.
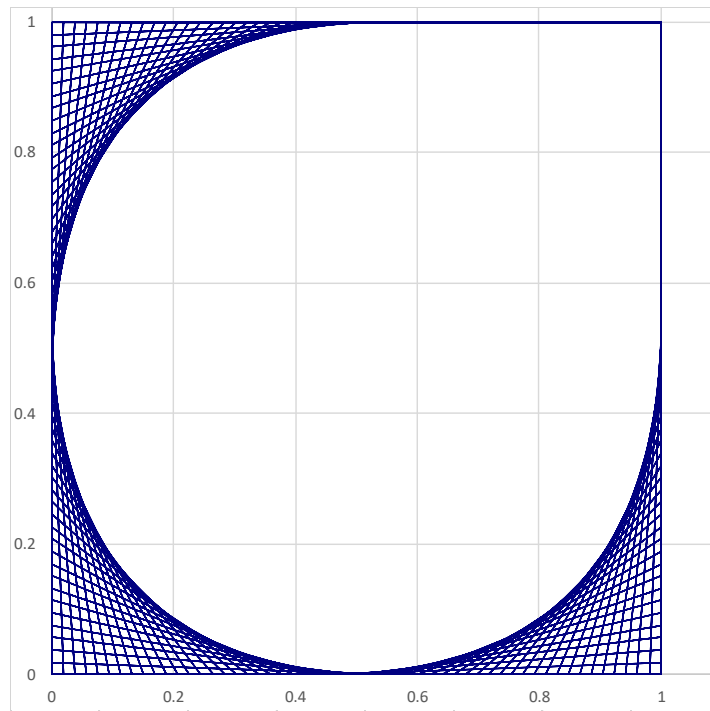

Figure 10: Image from sheet 7.A_trick with V = 5, S = 53, and P = 29.

Sheet 8.SomeShapes provides coordinates for various shapes in addition to V, S, and P values to try. Challenge questions are provided here as well. And finally, the next page is a one-page guide covering sections 1-3 above, created as a handout for classroom use.
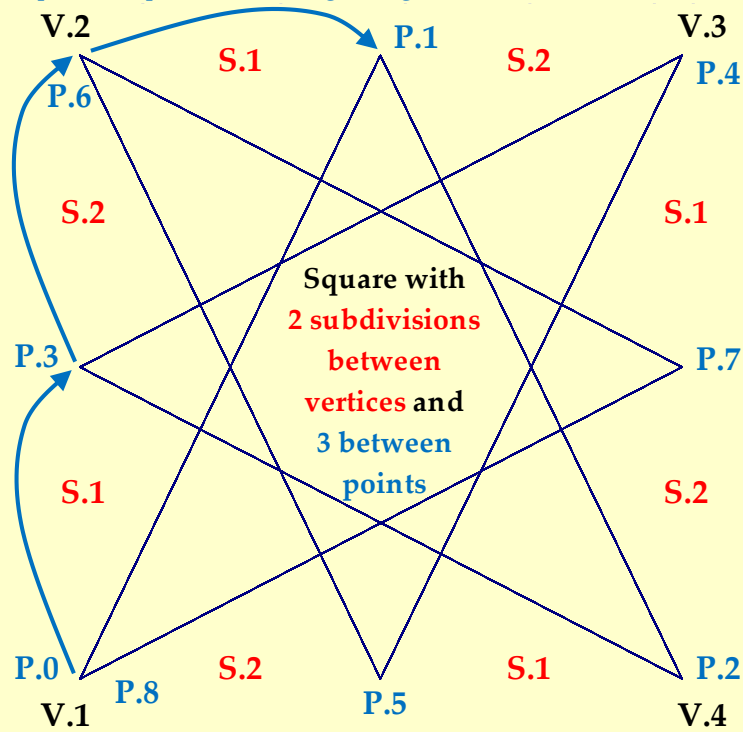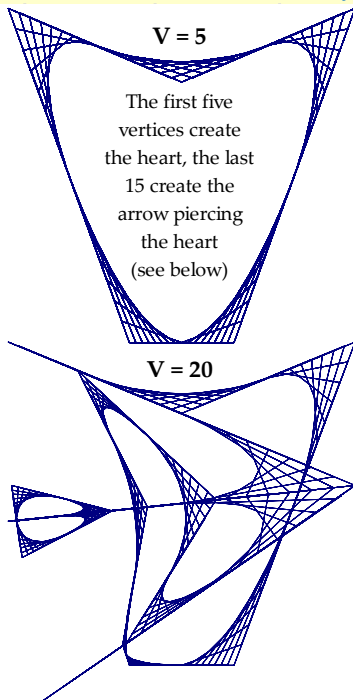
Handout for the Aestheometry.xlsx file

Aestheometry is the art of creating patterns and curved images using straight lines.

All figures graph line segments between consecutive (x, y) *Points* based on *Vertices* and number of *Subdivisions* between vertices. As you work through this file, you will learn to control *V*, *S*, and *P*.

Vertices, V **V** is the number of vertices used. Vertices provide the structure on which the image is created. All but two images on this page are based on a square so **V** = 4 in cell A1.

Subdivisions, S There are **S** subdivisions between each vertex. In the large image, **S** = 2 in cell B1.

Points, P Each image is based on drawing line segments from point to point. The first vertex is always point 0 (in the large image, **P.0** coincides with **V.1**). Subsequent points are **P** subdivisions beyond the previous point. In the large image, **P** = 3 in cell C1.

V = 5

The first five vertices create the heart, the last 15 create the arrow piercing the heart (see below)

V = 20

V.2   **S.1**   **P.1**   **S.2**   **V.3**

P.6   **P.4**

S.2   **S.1**

Square with 2 subdivisions between vertices and 3 between points

P.3   **P.7**

S.1   **S.2**

P.0   **S.2**   **S.1**   **P.2**

P.8   **P.5**

V.1   **V.4**

The images above vary *V, the Number of Vertices Used*, given **S** = 16 & **P** = 9.

Different images emerge based on different underlying vertices.

Left to Middle below varies *P, the Number of Subdivisions between Points*, given **V** = 4 & **S** = 5.

Middle to Right below varies *S, the Number of Subdivisions between Vertices*, given **V** = 4 & **P** = 9.

S = 5
P = 7

S = 5
P = 9

S = 16
P = 9