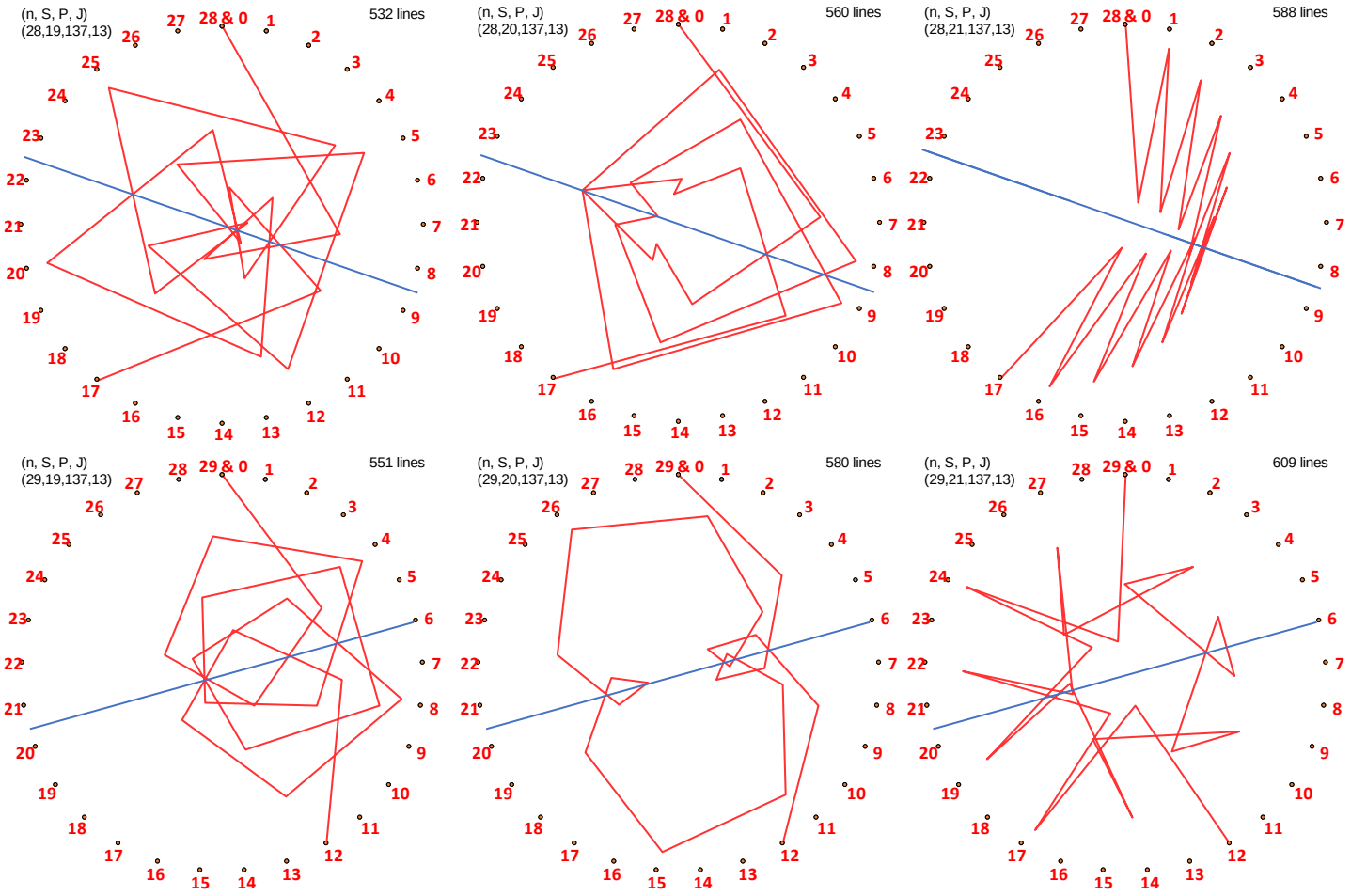
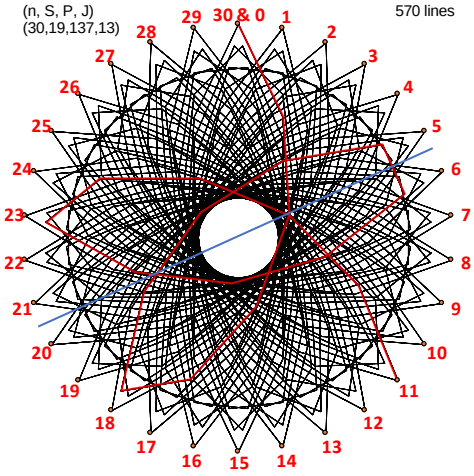
7.4. Symmetry across a Cycle

One of the points made in Section 7.3 was that the level changes are symmetric about ***S***/2. A closer look suggests that level changes are symmetric about ***C***/2 where ***C*** is the number of lines in a cycle (recall from Section 5.1, ***C*** = ***S***/GCD(***S***,***P***)). The vertical symmetry discussed in Section 6.3.1 is based on noting that the end of the last cycle looks like the start of the first cycle, and so forth for the second line and the second to last line. Putting these ideas together we see that a cycle is symmetric about its midpoint. This symmetry extends beyond symmetry of Levels; it can be described in terms of a line of symmetry that must always exist in the cycle. That line of symmetry is from the point ***E***/2 to (***n***+***E***)/2 where ***E*** is the end of the first cycle (Section 5.2). This **line of symmetry** is overlaid on the **first cycle** in each image below.

All images share a common ***P*** = 137 and ***J*** = 13; ***n*** varies by row and ***S*** varies by column. Since ***P*** is prime, ***C*** = ***S***.



Note: The location of the **blue line** varies by row but not column because ***E*** depends on ***n*** but not ***S*** when ***C*** = ***S***.



Even versus odd cycles. The middle column shows what happens when there are an even number of lines in the cycle. Notice that the **blue line** intersects at least one subdivision endpoint that is used to create the cycle. This is the end of line ***C***/2 (as noted above, ***C*** = ***S*** since ***P*** is prime). By contrast, one can clearly see that the **blue line** is perpendicular to the middle line of cycle when ***C*** is odd. This follows from the symmetry of levels across a cycle. When ***C*** is even, there is a single time that level is used but when ***C*** is odd, that level is seen twice in a row. *These middle lines can be used to create internal circles.*

The final image, with ***n*** = 30 but sharing ***S***, ***P***, and ***J*** with the left column, is provided to drive home this point. The image is shown with **first cycle in red**. Note the dark circle created by the midpoint lines. These lines are at Level 2,

just as they were when ***n*** = 28 and 29. By contrast, the midpoint level for the right two images is 5.