### 8.6. Kicking the Tires of Three Shape-Shifting Triangles

The Three Shape-Shifting Triangles image analyzed in Section $8.4,(\boldsymbol{n}, \mathbf{S}, \boldsymbol{P}, \boldsymbol{J})=(30,19,163,13)$, is one of a handful of images that has acted as a springboard to deeper understanding of the intricacies of ESA images. The seminal attribute of this image is that it is single-step of length 7 as defined in Section 8.5.1. (Other attributes are highlighted in yellow below.)

This follows because $\boldsymbol{n} \cdot \boldsymbol{S}=570$ and $7 \cdot \boldsymbol{P}=7 \cdot 163=1141=2 \cdot 570+1$. [MA. Using Chapter 24,7 and $\boldsymbol{P}$ are MMI MOD $\boldsymbol{n} \cdot \boldsymbol{S}$.]
The interesting thing to note about the above calculation is that it does NOT depend on $J$. Each $(n, S, P, J)=(30,19,163, J)$ image for $1 \leq J<n / 2$ will be single-step of length 7 . The image will not remain 570 lines long as $J$ varies because VCF > 1 for many of these values of $J$. The number of lines in the image is $n \cdot S /$ VCF because SCF $=1$ (since 163 is prime). The table below provides a summary of the 14 images that emerge as $J$ varies from 1 to 14 . The three images below focus on $J=9$.

How is the image filled in? The 7 -line sub-image will ALWAYS appear to rotate clockwise because the $7^{\text {th }}$ endpoint is just to the right and below the top (since $\boldsymbol{J}<\boldsymbol{n} / 2$ ). At left is the image, $\boldsymbol{k}=7$ shows the $1^{\text {st }}$ step, and $\boldsymbol{k}=19$ shows the $1^{\text {st }}$ cycle.


The blue highlighted row of the table shows attributes of the above image. One can see the open 7,3-star sub-image in the middle panel. This 7,3-star rotates $\circlearrowright$ but note that the image is filled in in a 1-time around $\cup$ fashion (see Section 5.2) because the first used vertex (at the end of the $k=191^{\text {st }}$ cycle in the right panel) is 27 (and VCF $=3$ so this is the first $U$ vertex used). Note also the 10,3-star vertex frame is visible in each panel, but most especially in the left image.

Create a video of these 14 images. By setting Drawn Lines $=7$ in the web version FCLD mode and clicking on $J$ so you can change $J$ using the up or down arrows, you can show each of these 14 images sequentially. Change $\boldsymbol{J}$ by 1 each time the image gets completed and if you extend $\boldsymbol{J}$ from 16 to 29 you will see the same images drawn in reverse. Below is $\boldsymbol{J}=7$.

A 5SST with DL = 11. On a related note, check out $(29,13,137,13)$.

| J | VCF | Vertices used, $V_{u}$ | Singlestep subimage | VF of <br> final <br> image | Number of Lines $L=19 V_{u}$ | Number of steps^ | First cycle ends at | Times around for image |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 30 | 7,2-star | 30-gon | 570 | 81 | 13 | $\bigcirc 13$ |
| 2 | 2 | 15 | 7,3-star | 15-gon | 285 | 41 | 26 | $\cup 2$ |
| 3 | 3 | 10 | 7-gon | 10-gon | 190 | 27 | 9 | $\cup 3$ |
| 4 | 2 | 15 | 7-gon | 15,2-star | 285 | 41 | 22 | $\cup 4$ |
| 5 | 5 | 6 | 7,3-star | 6-gon | 114 | 16 | 5 | $\cup 1$ |
| 6 | 6 | 5 | 7,2-star | 5-gon | 95 | 14 | 18 | $\cup 2$ |
| 7 | 1 | 30 | 7,3-star | 30,7-star | 570 | 81 | 1 | $\cup 1$ |
| 8 | 2 | 15 | 7,2-star | 15,4-star | 285 | 41 | 14 | $\cup 7$ |
| 9 | 3 | 10 | 7,3-star | 10,3-star | 190 | 27 | 27 | $\cup 1$ |
| 10 | 10 | 3 | 7-gon* | 3-gon | 57 | 8 | 10 | $\cup 1$ |
| 11 | 1 | 30 | NC 7-gon~ | 30,11-star | 570 | 81 | 23 | $\cup 7$ |
| 12 | 6 | 5 | 7,3-star | 5,2-star | 95 | 14 | 6 | $\cup 1$ |
| 13 | 1 | 30 | 3SST | 30,13-star | 570 | 81 | 19 | $\cup 11$ |
| 14 | 2 | 15 | 7,3-star | 15,7-star | 285 | 41 | 2 | $\cup 1$ |
| *appears as a 5-gon or a 6-gon due to 1 to 2 collinear sides across a cycle. |  |  |  |  |  |  |  |  |
| $\sim_{\text {non-convex }} 7$-gon. |  |  |  | ${ }^{\wedge}$ Calculated as $\operatorname{ROUND}(L / 7,0)$ |  |  |  |  |



