8.6. Kicking the Tires of Three Shape-Shifting Triangles

The *Three Shape-Shifting Triangles* image analyzed in Section 8.4, (n, S, P, J) = (30, 19, 163, 13), is one of a handful of images that has acted as a springboard to deeper understanding of the intricacies of ESA images. *The seminal attribute of this image is that it is single-step of length 7* as defined in Section 8.5.1. (Other attributes are highlighted in yellow below.)

This follows because *n*·*S* = 570 and 7·*P* = 7·163 = 1141 = 2·570 + 1. [MA. Using Chapter 24, 7 and *P* are MMI MOD *n*·*S*.]

The interesting thing to note about the above calculation is that it does NOT depend on J. Each (n, S, P, J) = (30, 19, 163, J)image for $1 \le J < n/2$ will be single-step of length 7. The image will not remain 570 lines long as J varies because VCF > 1 for many of these values of J. The number of lines in the image is $n \cdot S/VCF$ because SCF = 1 (since 163 is prime). The table below provides a summary of the 14 images that emerge as J varies from 1 to 14. The three images below focus on J = 9.

How is the image filled in? The 7-line sub-image will ALWAYS appear to rotate clockwise because the 7th endpoint is just to the right and below the top (since J < n/2). At left is the image, k = 7 shows the 1st step, and k = 19 shows the 1st cycle.



The blue highlighted row of the table shows attributes of the above image. One can see the **open 7,3-star sub-image** in the middle panel. This **7,3-star rotates** \bigcirc but note that the image is filled in in a 1-time around \bigcirc fashion (see Section 5.2) because the first used vertex (at the end of the $k = 19 \, 1^{\text{st}}$ cycle in the right panel) is 27 (and VCF = 3 so this is the first \bigcirc vertex used). Note also the 10,3-star vertex frame is visible in each panel, but most especially in the left image.

Create a video of these 14 images. By setting *Drawn Lines* = 7 in the web version FCLD mode and clicking on **J** so you can change **J** using the up or down arrows, you can show each of these 14 images sequentially. Change **J** by 1 each time the image gets completed and if you extend **J** from 16 to 29 you will see the same images drawn in reverse. Below is J = 7.

A 5SST with DL = 11. On a related note, check out (29,13,137,13).

		Vertices	Single-	VF of	Number		First	Times
		used,	step	final	of Lines	Number	cycle	around
J	VCF	Vu	subimage	image	$L = 19V_u$	of steps^	ends at	for image
1	1	30	7,2-star	30-gon	570	81	13	Č 13
2	2	15	7,3-star	15-gon	285	41	26	ٿ 2
3	3	10	7-gon	10-gon	190	27	9	Č3
4	2	15	7-gon	15,2-star	285	41	22	ن 4
5	5	6	7,3-star	6-gon	114	16	5	じ1
6	6	5	7,2-star	5-gon	95	14	18	ڻ 2
7	1	30	7,3-star	30,7-star	570	81	1	ひ1
8	2	15	7,2-star	15,4-star	285	41	14	ひ7
9	3	10	7,3-star	10,3-star	190	27	27	ٿ 1
10	10	3	7-gon*	3-gon	57	8	10	ひ1
11	1	30	NC 7-gon~	30,11-star	570	81	23	ن 7
12	6	5	7,3-star	5,2-star	95	14	6	ひ1
13	1	30	3SST	30,13-star	570	81	19	ڻ 11
14	2	15	7,3-star	15,7-star	285	41	2	ひ1
*appears as a 5-gon or a 6-gon due to 1 to 2 collinear sides across a cycle.								
~non-convex 7-gon.				^Calculated as ROUND(L/7,0)				

