# Zayla Introduces Teachers to the *Twilight Zone*
Or: *You **Can** Teach an Old Dog New Tricks*

Background: I have been writing *Playing with Polygons* for more than two years and wrote a paper about spirals for **Bridges** in 2021. As a result, I was very surprised to find a 4th grader who had found something that I could not explain using the spirals web model developed by Liam Myles. My First Year Seminar students were doing a community outreach project when a couple of them came up and asked me to come see what Zalya was getting from the spirals model.

The images she shows us was something that reminded me of optical illusions from TV programs I had seen in my childhood like "The Twilight Zone." Of course, this is such an obscure reference that it needs to be explained so it is not that good … on the other hand, I do not have a better tag line, so I am going with that.

The issue: Spirals created from single jumps have lines that do not intersect. With multiple jumps, lines intersect, and these intersections create areas of lighter and darker background. Despite this, there are areas that are darker and lighter within certain images and these darker and lighter areas create swirls. It turns out that the swirls are formed from a simple pattern in the square case and this pattern is most effectively understood by examining when swirls are not present. The six images below show 10, 8, 5, 4, 2, and 1 light areas, **L**, based on r = 100, 125, 200, 250, 500, and 1000.
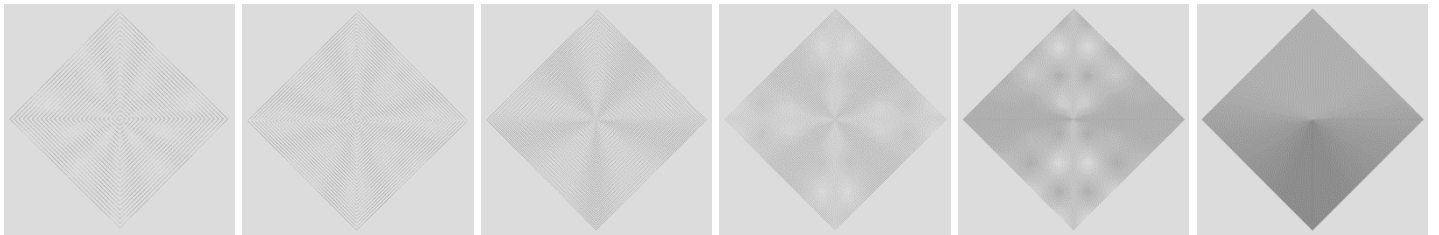
Note that $r \cdot L$ = 1000. Call these the **No-Swirl** values of r_{NS}. As r increases from each of these values, clockwise swirls to the center are created and as r decreases from these values, counterclockwise swirls to the center are created.

How many rotations does the swirl make in going to the center? With the square, each change in r creates a quarter turn rotation. This is consistent with the basic structure of spiral creation as noted in annotated r. If the distance from a no swirl value is 4, a single swirl of the light and dark portion of the image occurs. This is shown for r_{NS} = 200, 250, 500, and 1000. The top row shows r_{NS}+4 clockwise swirls and the bottom shows r_{NS}-4 counterclockwise swirls.
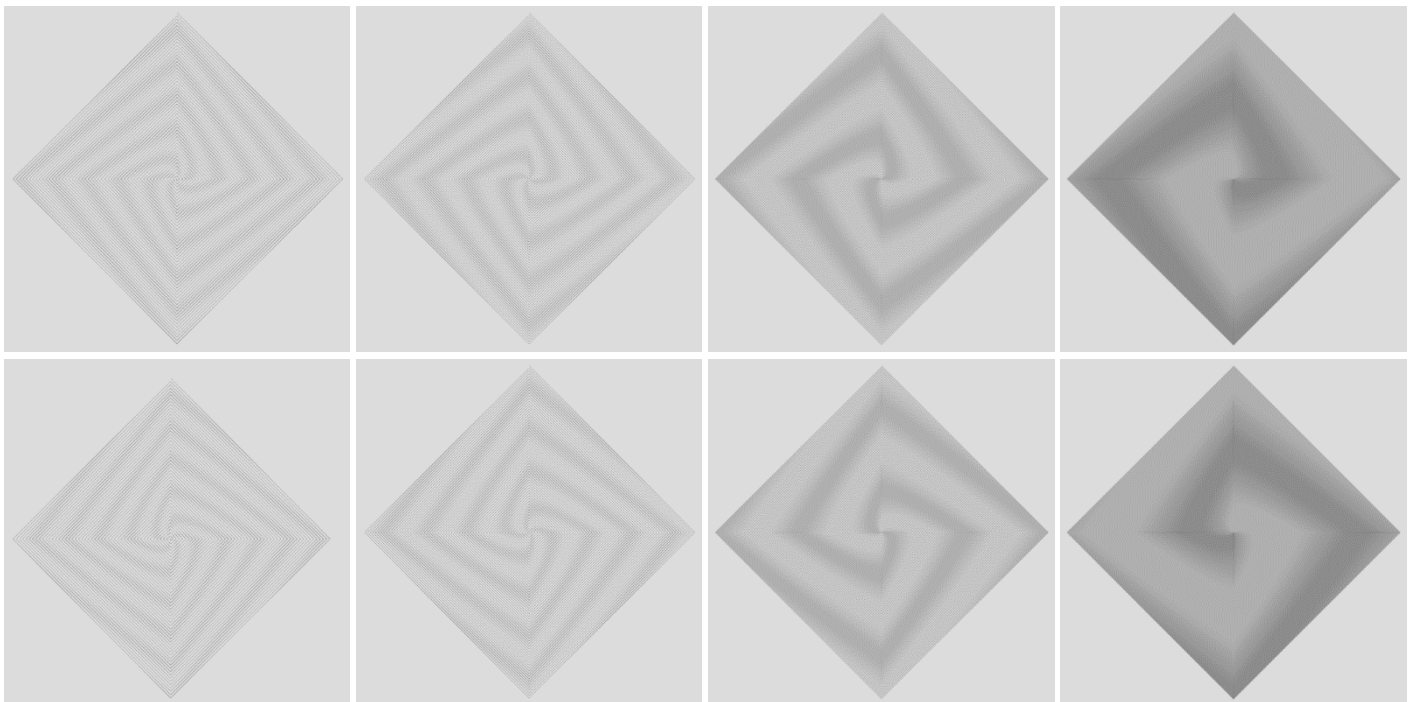
Each swirl above has a single rotation. The difference is that there is a swirl associated with each of the lighter areas.

How many swirls are there? When *r* diverges from a ***No-Swirl*** value, each of the lighter areas begins to swirl (of course, so does each of the darker areas as well since the swirl is created by the light/dark edges). It may help to think of mixing paint, or stirring together a recipe in which two different things are being mixed.

What creates the light and dark areas? The number of light wedges in the six no swirl images in the first row were based on values of 1000/*r* that produced whole number answers. Put another way, each *r* was a factor of 1000 greater than or equal to 100. The optical illusion requires enough lines to be present. The *r* = 50 image to the right shows that it is possible to discern 5 lighter areas on each of the four sides or *L* = 20 (the first 5 are labelled). Four are interior parts of the line segment and the 5th is at each vertex.

It is also possible in this instance to note that individual lines in each quadrant appear to have darker and lighter portions. Some portions of each line appear to be wider than others. I am not enough of a computer programmer to know why this is the case but clearly this is what is causing the optical illusion.

Static: The transition between swirling clockwise and counterclockwise images. As r increases past a No-Swirl image clockwise swirls occur but eventually those increases approach the next No-Swirl image from below and counterclockwise swirls occur. Unsurprisingly, the transition occurs about halfway between numbers, or more specifically, when the remainder of 1000/*r* close to 0.5. The image to the right shows *r* = 133, the transition between *L* = 8 and 7. Note that the dark and light portions of lines are staggered. This is most obvious at the 4 vertices of the spiral. Every other line is darker at each corner.

What about inexact *No-Swirl* values? It is not necessary for *r* to be a factor of 1000 to obtain ***No-Swirl*** images. The number of light areas *L* in the first row was: 10, 8, 5, 4, 2, and 1 so it seems natural to see if you can get 3, or 6, or 7, or 9 areas as well. In fact, what is required is that 1000/*r* is close to a whole number. There are 3 when *r* = 333; 6 when *r* = 167; 7 when *r* = 143; and 9 when *r* = 111.

Four versions of 12 layer clockwise swirls. Each image has 12 layers to the center, but each is based on a different number of interleaved swirls. From left to right (with links to each image): *r* = 262 = 250+3·4 has 4 swirls with 3 rotations; *r* = 349 = 333+4·4 has 3 swirls with 4 rotations; *r* = 524 = 500+6·4 has 2 swirls with 6 rotations; *r* = 1048 = 1000+12·4 has 1 swirl with 12 rotations. This difference is easiest to see by comparing the centers (especially the last two).
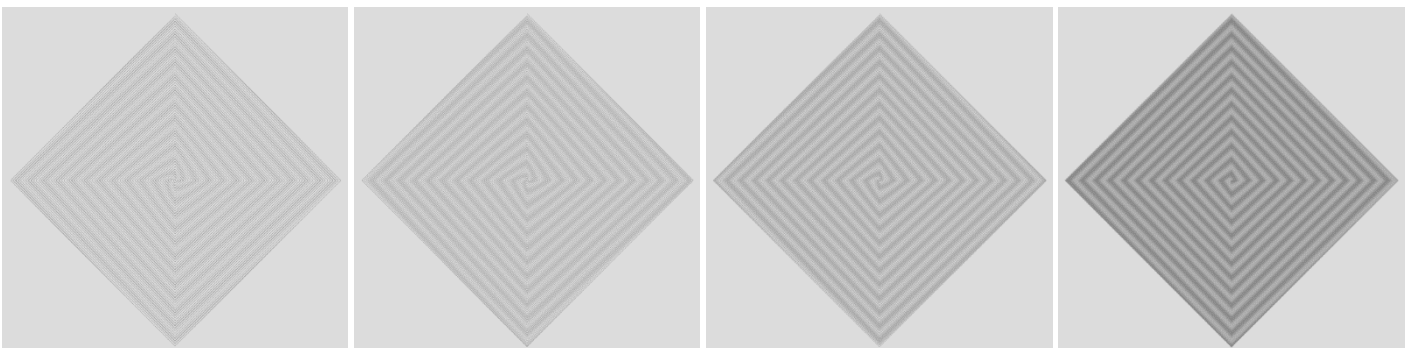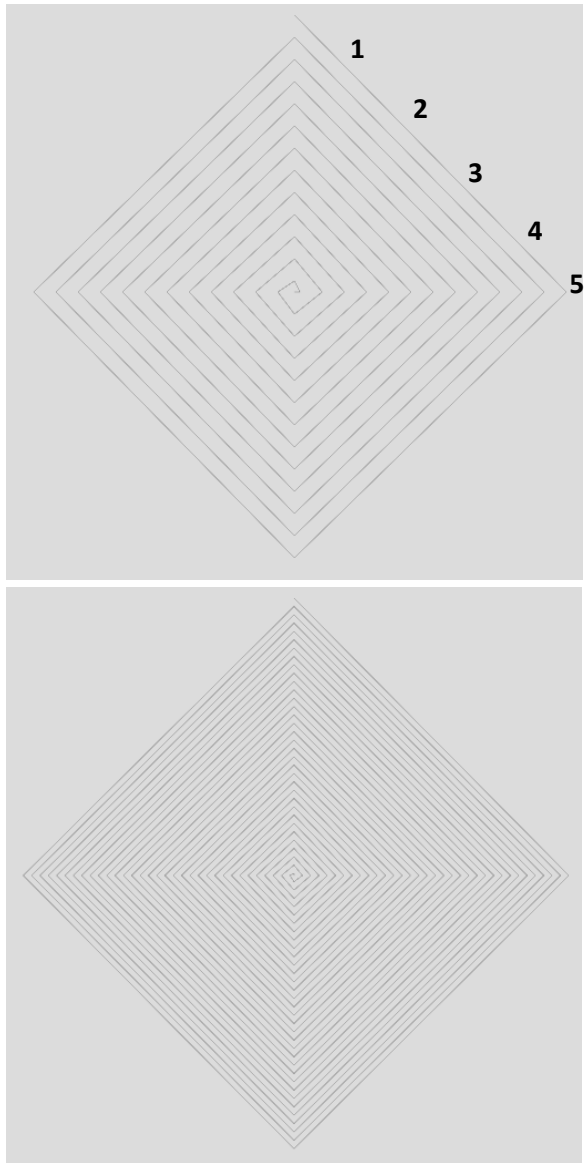
[MA. Note that the last line, the one drawn to the center given *n* = 4, depends on *r* MOD 4. If 0 = *r* MOD 4, it is on the -x axis. If 1 = *r* MOD 4, it is on the +y axis. If 2 = *r* MOD 4, it is on the +x axis. If 3 = *r* MOD 4 it is on the -y axis.]