21.3. What is the Product of Two Numbers that Differ by an Even Amount? (This looks at *times table* patterns along diagonals.)

Here is a different way to look at a *times table*. It is provided to highlight a pattern in numbers that differ by an even amount. It is based on a formula used in a couple of places in ESA called the **Difference between Squares** formula. That formula is $(x+y)\cdot(x-y) = x^2 - y^2$. This works for any x and y, not just whole numbers.

Let $a - b = 2k$ where a, b, and k are whole numbers. This means that the difference between a and b is an even number.

The number $c = b+k$ is halfway between a and b but so is $c = a-k$. Regrouping both we have $a = c+k$ and $b = c-k$.

The product of a and b is thus: $a\cdot b = (c+k)\cdot(c-k)$

Distributing the right hand side we obtain: $a\cdot b = (c+k)\cdot c - (c+k)\cdot k$

Distributing once again we obtain: $a\cdot b = c^2 + k\cdot c - c\cdot k - k^3$

Cancelling common terms: $a\cdot b = c^2 - k^2$

How does this relate to the highlighted cells? Look at the numbers inside each red oval.

c **Yellow** cells are perfect squares. These are the values of c, the center number.

$k=1$ **Green** cells are 1 less in one direction, 1 more in the other. So, subtract 1.

$k=2$ **Blue** cells are 2 less in one direction, 2 more in the other. So, subtract $4 = 2^2$.

$k=3$ **Tan** cells are 3 less in one direction, 3 more in the other. So, subtract $9 = 3^2$.

This same pattern works regardless of where the center c is located!

Look to the adjoining cells at left above or at right below.

y x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	…	…	…	50	…				
																										-3^2						
																									-9		-2^2					
																								2491		-4		-1^2				$k=3$
																									2496		-1			$k=1$		
…																										2499		c^2			$k=2$	
50																											2500		-1			
…																										…		2499		-4		
…																									…				2496		-9	
…																								…						2491		
23																	391						…									
22																		396				…										
21																			399		…											
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400												
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399											
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360		396										
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340			391									
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320												
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300												
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280												
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260												
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240												
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220												
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200												
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180												
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160												
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140												
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120												
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100												
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80												
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60												
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40												
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20												

This pattern can be extended beyond $k = 3$. In the end, half of all multiplication values can be calculated this way.

This can be used to excite young learners about playing with numbers once they know about multiplication. Here is how.

1. Teach perfect squares of "round" numbers.
   10, 15, 20, 30, …, depending on age level.
2. Ask them multiplication questions that "surround" those numbers.
3. For example, most would work at 17·23
   But they have no problem subtracting 23-17
   The result, 6, divided by 2 is 3, $3^2 = 9$.
   The midpoint is 20.
   The square of 20 is easy … 4 with 2 zeros.

So **17·23** = 400 - 3^2 = 400 - 9 = **391**.