How many Images are in a *Sequence Player* Sequence?

The *Sequence Player* mode provides a wonderful way to scroll through images based on a choice of *n*, *S*, and *J*. Two natural questions are: A) How many images are in the sequence? B) How many of these images are distinct? We examine these questions in a couple of steps based on single jump images.

1. The first thing to understand is the restriction placed on images shown in *Sequence Player* mode. Each image (except, perhaps the starting image since initial *P* is set by the user) has SCF = 1. This restriction avoids the "flickering" that occurs when you scroll through *P* one at a time when SCF > 1. This flickering is discussed in E10.2.1 and in ESA Video 5.

2. Image sequences are symmetric about *n*/VCF·*S*/2. The images in the back half of the sequence are the same as the front half, but they are seen in reverse order. This is most easily seen at the two ends where the curves in the curved-tip stars get larger in the front half, but get smaller in the back half, and at the middle where the second porcupine image initiates the second half back-down of images. Beyond that, it sometimes appears that there are different images in the front half and the back half, but they are the same, only seen in different order. Here is an analogy: Imagine you have a short video of a plane landing then play that video in reverse – the two versions do not have the same "feel," and one might imagine that they were not created from the same underlying video frames.

3. Because of 1) and 2), the number of images in a *Sequence Player* sequence is two times the number of numbers *P* that are coprime to *n*/VCF·*S*/2. If VCF = 1 then *P* must be coprime to factors of *n* and *S*. One can easily obtain this number by creating a modified form of the *Sieve of Eratosthenes*, E21.1.2.

The modification is to use *n* columns rather than 10 and R = ROUND(*S*/2) rows.

Note that all values *P* in a column that is a factor of *n* is also a factor of *P*+*n*. These are the columns highlighted in yellow in this table. If *S* is prime, multiples of *S* that are not factors of *n* are highlighted in green. The remaining (unhighlighted) numbers are coprime to *n*·*S*.

This is shown for the default image on the *Sequence Player* page, (12,41,*P*,5). The image starts at *P* = 1 but cycles from there through all the 160 SCF = 1 values of *P* in the order 1, 5, 7, 11, 13, … , 241, 245, 247 = 492-245, 251 = 492-241, … , 491 = 492-1.

One can see these individual images by slowing down the speed dramatically (set Speed to 1000 rather than 100) or by scrolling through *P* from 1 to 492 manually using this link.

Row R (R≤ROUND(S/2))					12 =n		41 =S		80 SCF = 1 < nS /2			
R	41	82	123	164	205	246	= ROW1*S for ROW1 ≤ n/2					
1	1	2	3	4	5	6	7	8	9	10	11	12
2	13	14	15	16	17	18	19	20	21	22	23	24
3	25	26	27	28	29	30	31	32	33	34	35	36
4	37	38	39	40	41	42	43	44	45	46	47	48
5	49	50	51	52	53	54	55	56	57	58	59	60
6	61	62	63	64	65	66	67	68	69	70	71	72
7	73	74	75	76	77	78	79	80	81	82	83	84
8	85	86	87	88	89	90	91	92	93	94	95	96
9	97	98	99	100	101	102	103	104	105	106	107	108
10	109	110	111	112	113	114	115	116	117	118	119	120
11	121	122	123	124	125	126	127	128	129	130	131	132
12	133	134	135	136	137	138	139	140	141	142	143	144
13	145	146	147	148	149	150	151	152	153	154	155	156
14	157	158	159	160	161	162	163	164	165	166	167	168
15	169	170	171	172	173	174	175	176	177	178	179	180
16	181	182	183	184	185	186	187	188	189	190	191	192
17	193	194	195	196	197	198	199	200	201	202	203	204
18	205	206	207	208	209	210	211	212	213	214	215	216
19	217	218	219	220	221	222	223	224	225	226	227	228
20	229	230	231	232	233	234	235	236	237	238	239	240
21	241	242	243	244	245	246	These numbers are > nS/2					
	20	coprime per			20		20		column			20

Composite S. When **S** is composite one must eliminate the prime factors of **S** sequentially. This is shown for an example that uses the first four prime numbers: <u>n = 15, S = 14</u>. The first iteration removes 49 multiples of 3 and 5, the factors of **n** (yellow), the next round removes 28 multiples of 2 (green), and the final round removes 4 multiples of 7 (blue). The 24 = 105-49-28-4 numbers remaining are coprime to 210, the number of lines in each image.

(Note that if there were more than two prime factors of **S**, additional rounds would need to be added.)

If you click **Play Sequence**, the resulting 48 image sequence is nowhere near as smooth as the earlier image sequence analyzed because there are large jumps between some successive SCF = 1 **P** values.

By contrast, the final table shows the benefit of having **S** be prime (15,13,**P**,4) even though that prime is a smaller number. Each image has 195 lines, but there are twice as many images in the image sequence, despite a smaller **S**.

An even smoother solution is to use a single prime. If **n** and **S** are both powers of 2, there are **nS**/2 images for odd **J** since each **P** is odd (so there are 512 images for (32,32,**P**,15). More generally, if **a** is prime, $n = a^b$ and $S = a^c$, there are $(a-1)a^{(b+c-1)}$ images in the image sequence as long as GCD(**J, a**) = 1 because 1 to **a**-1 are coprime to **a**. Thus, both 625 line image sequences (25,25,**P**,12) and (5,125,**P**,2) have 500 images.

Row R (R≤ROUND(S /2))		15 =n		14 =S		210 =nS					24 SCF = 1 < nS/2			
1	1	2	**3**	4	**5**	**6**	7	8	**9**	**10**	11	**12**	13	14	**15**
2	16	17	**18**	19	**20**	**21**	22	23	**24**	**25**	26	**27**	28	29	**30**
3	31	32	**33**	34	**35**	**36**	37	38	**39**	**40**	41	**42**	43	44	**45**
4	46	47	**48**	49	**50**	**51**	52	53	**54**	**55**	56	**57**	58	59	**60**
5	61	62	**63**	64	**65**	**66**	67	68	**69**	**70**	71	**72**	73	74	**75**
6	76	77	**78**	79	**80**	**81**	82	83	**84**	**85**	86	**87**	88	89	**90**
7	91	92	**93**	94	**95**	**96**	97	98	**99**	**100**	101	**102**	103	104	**105**

R	Round 2: Remove 7·7 = 49 **P** containing factors of 15 & highlight multiples of 2													
1	1	**2**		**4**			7	**8**			11		13	**14**	
2	**16**	17		19			**22**	23			**26**		**28**	29	
3	31	**32**		**34**			37	**38**			41		43	**44**	
4	**46**	47		49			**52**	53			**56**		**58**	59	
5	61	**62**		**64**			67	**68**			71		73	**74**	
6	**76**	77		79			**82**	83			**86**		**88**	89	
7	91	**92**		**94**			97	**98**			101		103	**104**	

R	Round 3: Remove 28 even values and highlight multiples of 7													
1	1						**7**				11		13		
2		17		19				23						29	
3	31						37				41		43		
4		47		**49**				53						59	
5	61						67				71		73		
6		**77**		79				83						89	
7	**91**						97				101		103		

R	Remove 4 multiple of 7 values. The remaining 24 **P** are coprime to 210.													
1	1										11		13		
2		17		19				23						29	
3	31						37				41		43		
4		47						53						59	
5	61						67				71		73		
6				79				83						89	
7							97				101		103		

Row R (R ≤ ROUND(S/2))		15 =n		13 =S		195 =nS					48 SCF = 1 < nS/2			
R	**13**	**26**	**39**	**52**	**65**	**78**	**91**	= ROW1*S for ROW1 ≤ n /2						
1	1	2	**3**	4	**5**	**6**	7	8	**9**	**10**	11	**12**	13	14	**15**
2	16	17	**18**	19	**20**	**21**	22	23	**24**	**25**	26	**27**	28	29	**30**
3	31	32	**33**	34	**35**	**36**	37	38	**39**	**40**	41	**42**	43	44	**45**
4	46	47	**48**	49	**50**	**51**	52	53	**54**	**55**	56	**57**	58	59	**60**
5	61	62	**63**	64	**65**	**66**	67	68	**69**	**70**	71	**72**	73	74	**75**
6	76	77	**78**	79	**80**	**81**	82	83	**84**	**85**	86	**87**	88	89	**90**
7	**91**	92	**93**	94	**95**	**96**	97	These numbers are greater than nS/2.						