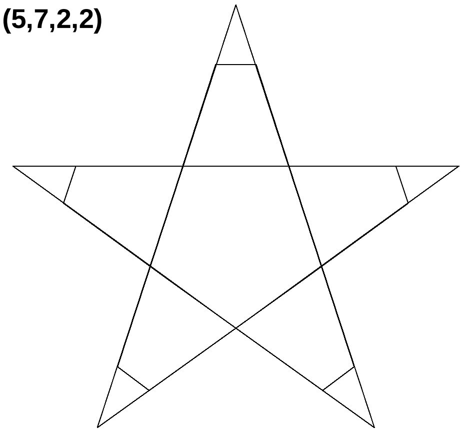
Truncated Stars and *Stars with Eyelets*

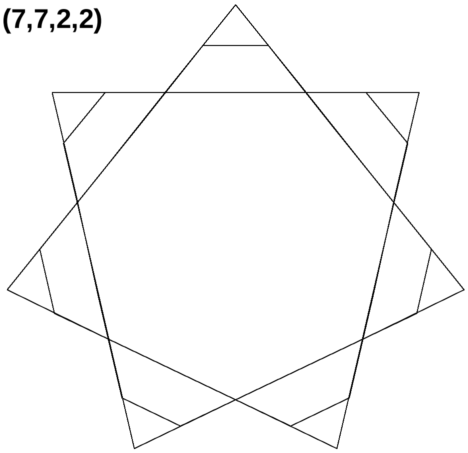
One of the simplest images in **ESA** is a star that has lines equidistant from each star's points. If ***n*** and ***S*** are odd, ***J*** is coprime to ***n***, and ***P*** = 2, such an image is the result. The top row shows three such images.

The next two rows show **E16** double jump models where the first jump is 1 and the second jump is chosen to create a "version" of the top row. To focus on the underlying structure, each image shows the vertex frame since ***S*** = ***P*** = 1. The second row shows tilted versions of the image above with the star-tips *truncated* (or removed). The third row shows the same stars suspended on the interior of the image with triangular *eyelets* attached to each star-tip.

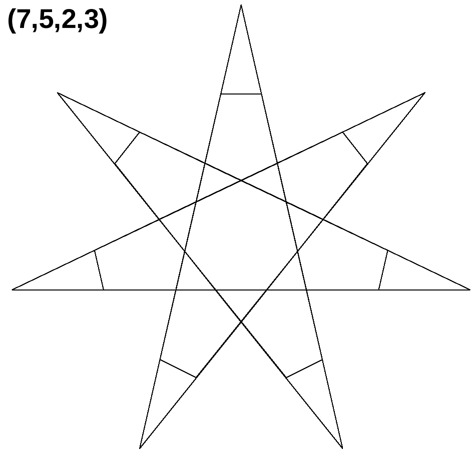
(5,7,2,2)



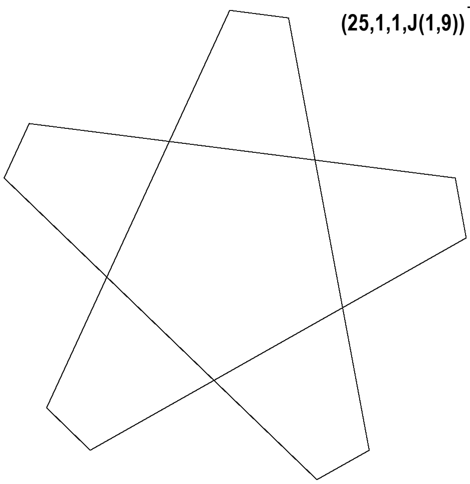
(7,7,2,2)



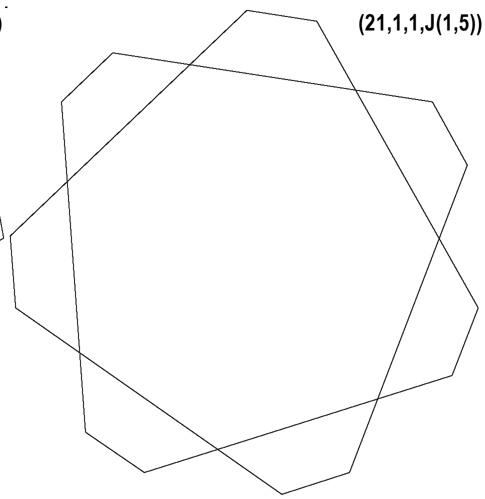
(7,5,2,3)



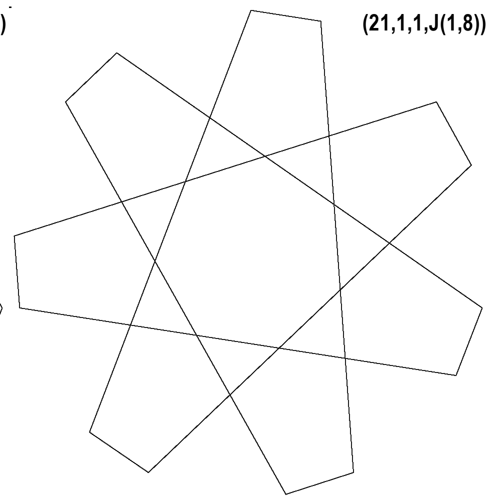
(25,1,1,J(1,9))



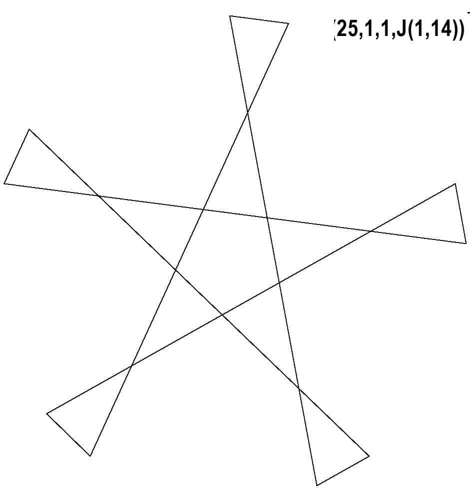
(21,1,1,J(1,5))



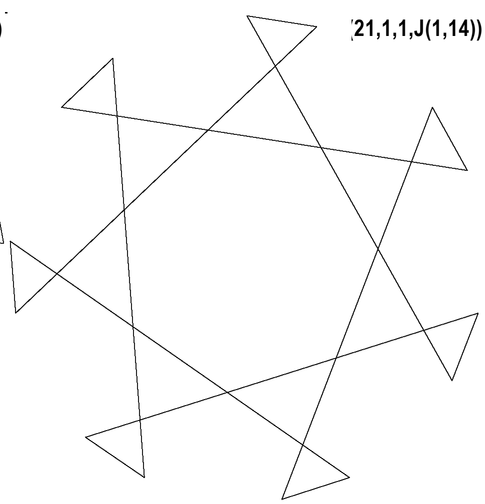
(21,1,1,J(1,8))



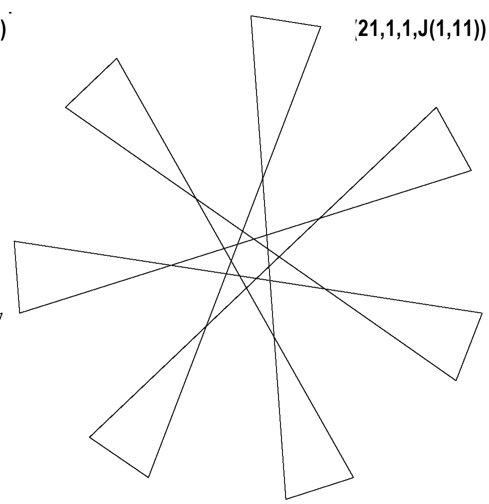
(25,1,1,J(1,14))



(21,1,1,J(1,14))



(21,1,1,J(1,11))



How much tilt do these images show? If you have an image which is either a truncated star or star with eyelets, let J_1 and J_2 be the two jumps. Let the minimum of (J_1, J_2, n-J_1, n-J_2) = m. The previous images all had m = 1 since J_1 = 1. In these examples, the “top” point of each star is centered between the top and vertex 1, or at an angle of 0.5/n·360° = 180/n°. More generally, the tilt will be 180·m/n°.

How to tilt the image in the opposite direction? If you switch (J_1, J_2) to (J_2, J_1) the image will tilt in the opposite direction. Subtracting from n and switching order does not change tilt so that (n-J_2, n-J_1) tilts the same as (J_1, J_2) (so (J_1, J_2) of (1,9) and (16,24) produce the same image given n = 25; the only difference is the order in which the static image is drawn).

How can we create similar images? Start with a G,r-star you like (where r < G/2). Multiply G by some factor k and use that multiple as n = G·k (25 = 5·5 or 21 = 7·3, so k = 5 on left and 3 on middle and right). This k is used to set J_1+J_2.

Truncated stars: Set J_1+J_2 = r·k if J_1+J_2 < n or J_1+J_2-n = (G-r)·k if J_1+J_2 > n. (From L-R in Row Two: 1+9 = 2·5; 1+5 = 2·3; and 1+8 = 3·3, or, to use the alternate version for the left middle image noted above, 16+24-25 = (5-2)·5.) Note also that if you change from (1,9) to (2,8) you have another truncated 5,2-star with larger star-tips missing.

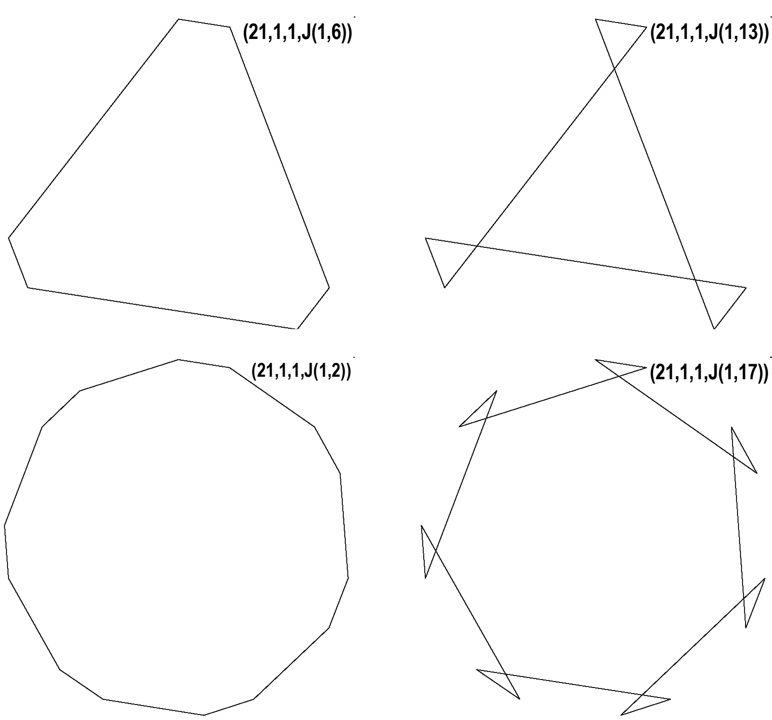
Stars with eyelets: Set J_1+J_2 = (G-r)·k if J_1+J_2 < n or J_1+J_2-n = r·k if J_1+J_2 > n. (From L-R in Row Three: 1+14 = (5-2)·5; 1+14 = (7-2)·3; and 1+11 = (7-3)·3. The alternate version for the left bottom image is (11,24) so, 11+24-25 = 2·5.) And, just like the truncated stars, change the size of the interior star and eyelets by changing from (1,14) to (2,13).

For both types of images, one can change the size of the truncated tips or eyelets by varying k and m.

Truncated Polygons and Polygons with Eyelets. This analysis extends beyond stars to polygons. Below are examples based on n = 21, of truncated triangles and 7-gons as well as triangles and 7-gons with eyelets obtained simply by varying J_2. The size of the G-gon or G-gram in each instance is easy to determine: it is just G = n/VCF where we recall from **E16.1** that in the double-jump setting, VCF = GCD(n, J_1+J_2).

A more general comparison. The table shows all 40 truncated and eyelet images for n = 60 and J_1 = 1. The same stars and polygons occur in ($_TJ_2$, $_EJ_2$) pairs with $_TJ_2$+$_EJ_2$ = n-2·J_1 if VCF > 1. In this instance, the jump level r of the G,r-star is given by r = (J_1+$_TJ_2$)/VCF. Given J_1 = 1, truncated stars have J_1+J_2 < n/2, and eyelet stars require J_1+J_2 > n/2 to create the eyelets.

It is worth watching the *Sequence Player* for each version, just by varying J_2. The porcupine images (with P = Lines/2 ± 1 or 2) are *Needle Fans*, see **E16.7**. One example is this 1230-Line 15,4 Needle fan (60,41,613,J(1,43)).



Truncated & Eyelet Images: n = 60, J_1 = 1

$_TJ_2$	VCF	G = 60/VCF	Common name		$_EJ_2$
2	3	20	20 -gon		56
3	4	15	15 -gon		55
4	5	12	12 -gon	dodecagon	54
5	6	10	10 -gon	decagon	53
7	4	15	15,2 -star		51
8	3	20	20,2 -star		50
9	10	6	6 -gon	hexagon	49
11	12	5	5 -gon	pentagon	47
13	2	30	30,7 -star		45
14	15	4	4 -gon	square	44
15	4	15	15,4 -star		43
17	6	10	10,3 -star		41
19	20	3	3 -gon	triangle	39
20	3	20	20,7 -star		38
21	2	30	30,11 -star		37
23	12	5	5,2 -star	pentagram	35
24	5	12	12,5 -star		34
25	2	30	30,13 -star		33
26	3	20	20,9 -star		32
27	4	15	15,7 -star		31
truncated	$_TJ_2$ + $_EJ_2$ = n-2				with eyelets