How many Images are in a *Sequence Player* Sequence?

The *Sequence Player* mode provides a wonderful way to scroll through images based on a choice of ***n***, ***S***, and ***J***. Two natural questions are: A) How many images are in the sequence? B) How many of these images are distinct? We examine these questions in a couple of steps based on single jump images.

1. The first thing to understand is the restriction placed on images shown in *Sequence Player* mode. Each image (except, perhaps the starting image since initial ***P*** is set by the user) has SCF = 1. This restriction avoids the "flickering" that occurs when you scroll through ***P*** one at a time when SCF > 1. This flickering is discussed in E10.2.1 and in ESA Video 5.

2. Image sequences are symmetric about ***n***/VCF·***S***/2. The images in the back half of the sequence are the same as the front half, but they are seen in reverse order. This is most easily seen at the two ends where the curves in the curved-tip stars get larger in the front half, but get smaller in the back half, and at the middle where the second porcupine image initiates the second half back-down of images. Beyond that, it sometimes appears that there are different images in the front half and the back half, but they are the same, only seen in different order. Here is an analogy: Imagine you have a short video of a plane landing then play that video in reverse – the two versions do not have the same "feel," and one might imagine that they were not created from the same underlying video frames.

3. Because of 1) and 2), the number of images in a *Sequence Player* sequence is two times the number of numbers ***P*** that are coprime to ***n***/VCF·***S***/2. If VCF = 1 then ***P*** must be coprime to factors of ***n*** and ***S***. One can easily obtain this number by creating a modified form of the *Sieve of Eratosthenes*, E21.1.2.

The modification is to use ***n*** columns rather than 10 and R = ROUND(***S***/2) rows.

Note that all values ***P*** in a column that is a factor of ***n*** is also a factor of ***P***+***n***. These are the columns highlighted in yellow in this table. If ***S*** is prime, multiples of ***S*** that are not factors of ***n*** are highlighted in green. The remaining (unhighlighted) numbers are coprime to ***n***·***S***.

This is shown for the default image on the *Sequence Player* page, (12,41,***P***,5). The image starts at ***P*** = 1 but cycles from there through all the 160 SCF = 1 values of ***P*** in the order 1, 5, 7, 11, 13, … , 241, 245, 247 = 492-245, 251 = 492-241, … , 491 = 492-1.

One can see these individual images by slowing down the speed dramatically (set Speed to 1000 rather than 100) or by scrolling through ***P*** from 1 to 492 manually using this link.

Row R (R≤ROUND(***S*** /2))					12	=***n***	41	=***S***	**80 SCF = 1 < *nS* /2**			
R	**41**	**82**	**123**	**164**	**205**	**246**	= ROW1*S for ROW1 ≤ ***n*** /2					
1	1	**2**	**3**	**4**	5	**6**	7	**8**	**9**	**10**	11	**12**
2	13	**14**	**15**	**16**	17	**18**	19	**20**	**21**	**22**	23	**24**
3	25	**26**	**27**	**28**	29	**30**	31	**32**	**33**	**34**	35	**36**
4	37	**38**	**39**	**40**	**41**	**42**	43	**44**	**45**	**46**	47	**48**
5	49	**50**	**51**	**52**	53	**54**	55	**56**	**57**	**58**	59	**60**
6	61	**62**	**63**	**64**	65	**66**	67	**68**	**69**	**70**	71	**72**
7	73	**74**	**75**	**76**	77	**78**	79	**80**	**81**	**82**	83	**84**
8	85	**86**	**87**	**88**	89	**90**	91	**92**	**93**	**94**	95	**96**
9	97	**98**	**99**	**100**	101	**102**	103	**104**	**105**	**106**	107	**108**
10	109	**110**	**111**	**112**	113	**114**	115	**116**	**117**	**118**	119	**120**
11	121	**122**	**123**	**124**	125	**126**	127	**128**	**129**	**130**	131	**132**
12	133	**134**	**135**	**136**	137	**138**	139	**140**	**141**	**142**	143	**144**
13	145	**146**	**147**	**148**	149	**150**	151	**152**	**153**	**154**	155	**156**
14	157	**158**	**159**	**160**	161	**162**	163	**164**	**165**	**166**	167	**168**
15	169	**170**	**171**	**172**	173	**174**	175	**176**	**177**	**178**	179	**180**
16	181	**182**	**183**	**184**	185	**186**	187	**188**	**189**	**190**	191	**192**
17	193	**194**	**195**	**196**	197	**198**	199	**200**	**201**	**202**	203	**204**
18	**205**	**206**	**207**	**208**	209	**210**	211	**212**	**213**	**214**	215	**216**
19	217	**218**	**219**	**220**	221	**222**	223	**224**	**225**	**226**	227	**228**
20	229	**230**	**231**	**232**	233	**234**	235	**236**	**237**	**238**	239	**240**
21	241	**242**	**243**	**244**	245	**246**	**These numbers are > *nS/2***					
	20	**coprime per**			**20**		**20**	**column**			**20**	

Composite *S*. When ***S*** is composite one must eliminate the prime factors of ***S*** sequentially. This is shown for an example that uses the first four prime numbers: ***n*** = 15, ***S*** = 14. The first iteration removes 49 multiples of 3 and 5, the factors of ***n*** (yellow), the next round removes 28 multiples of 2 (green), and the final round removes 4 multiples of 7 (blue). The 24 = 105-49-28-4 numbers remaining are coprime to 210, the number of lines in each image.

(Note that if there were more than two prime factors of ***S***, additional rounds would need to be added.)

If you click **Play Sequence**, the resulting 48 image sequence is nowhere near as smooth as the earlier image sequence analyzed because there are large jumps between some successive SCF = 1 ***P*** values.

By contrast, the final table shows the benefit of having ***S*** be prime (15,13,***P***,4) even though that prime is a smaller number. Each image has 195 lines, but there are twice as many images in the image sequence, 96, despite a smaller ***S***.

An even smoother solution is to use a single prime. If ***n*** and ***S*** are both powers of 2, there are ***nS***/2 images for odd ***J*** since each ***P*** is odd (so there are 512 images for (32,32,***P***,15). More generally, if ***a*** is prime, $\boldsymbol{n} = \boldsymbol{a}^{\boldsymbol{b}}$ and $\boldsymbol{S} = \boldsymbol{a}^{\boldsymbol{c}}$, there are $(\boldsymbol{a}\text{-}1)\cdot\boldsymbol{a}^{(b+c-1)}$ images in the image sequence as long as GCD(***J***, ***a***) = 1 because 1 to ***a***-1 are coprime to ***a***. Thus, both 625 line image sequences (25,25,***P***,12) and (5,125,***P***,2) have 500 images.

Row R (R≤ROUND(*S*/2))			15	=*n*	14	=*S*	210	=*nS*		**24 SCF = 1 < *nS/2***					
1	1	2	**3**	4	**5**	**6**	7	8	**9**	**10**	11	**12**	13	14	**15**
2	16	17	**18**	19	**20**	**21**	22	23	**24**	**25**	26	**27**	28	29	**30**
3	31	32	**33**	34	**35**	**36**	37	38	**39**	**40**	41	**42**	43	44	**45**
4	46	47	**48**	49	**50**	**51**	52	53	**54**	**55**	56	**57**	58	59	**60**
5	61	62	**63**	64	**65**	**66**	67	68	**69**	**70**	71	**72**	73	74	**75**
6	76	77	**78**	79	**80**	**81**	82	83	**84**	**85**	86	**87**	88	89	**90**
7	91	92	**93**	94	**95**	**96**	97	98	**99**	**100**	101	**102**	103	104	**105**

R	Round 2: Remove 7·7 = 49 ***P*** containing factors of 15 & highlight multiples of 2														
1	1	**2**		**4**			7	**8**			11		13	**14**	
2	**16**	17		19			**22**	23			**26**		**28**	29	
3	31	**32**		**34**			37	**38**			41		43	**44**	
4	**46**	47		49			**52**	53			**56**		**58**	59	
5	61	**62**		**64**			67	**68**			71		73	**74**	
6	**76**	77		79			**82**	83			**86**		**88**	89	
7	91	**92**		**94**			97	**98**			101		103	**104**	

R	Round 3: Remove 28 even values and highlight multiples of 7														
1	1						**7**				11		13		
2		17		19				23						29	
3	31						37				41		43		
4		47		**49**				53						59	
5	61						67				71		73		
6		**77**		79				83						89	
7	**91**						97				101		103		

R	Remove 4 multiple of 7 values. The remaining 24 ***P*** are coprime to 210.														
1	**1**										**11**		**13**		
2		**17**		**19**				**23**						**29**	
3	**31**						**37**				**41**		**43**		
4		**47**						**53**						**59**	
5	**61**						**67**				**71**		**73**		
6				**79**				**83**						**89**	
7							**97**				**101**		**103**		

This example decreases ***S*** by 1 but has twice as many SCF =1 images.

Row R (R ≤ ROUND(S/2))			15	=*n*	13	=*S*	195	=*nS*		**48 SCF = 1 < *nS/2***					
R	**13**	**26**	**39**	**52**	**65**	**78**	**91**	= ROW1**S* for ROW1 ≤ *n* /2							
1	1	2	**3**	4	**5**	**6**	7	8	**9**	**10**	11	**12**	**13**	14	**15**
2	16	17	**18**	19	**20**	**21**	22	23	**24**	**25**	**26**	**27**	28	29	**30**
3	31	32	**33**	34	**35**	**36**	37	38	**39**	**40**	41	**42**	43	44	**45**
4	46	47	**48**	49	**50**	**51**	**52**	53	**54**	**55**	56	**57**	58	59	**60**
5	61	62	**63**	64	**65**	**66**	67	68	**69**	**70**	71	**72**	73	74	**75**
6	76	77	**78**	79	**80**	**81**	82	83	**84**	**85**	86	**87**	88	89	**90**
7	**91**	92	**93**	94	**95**	**96**	97	**These numbers are greater than *nS/2*.**							