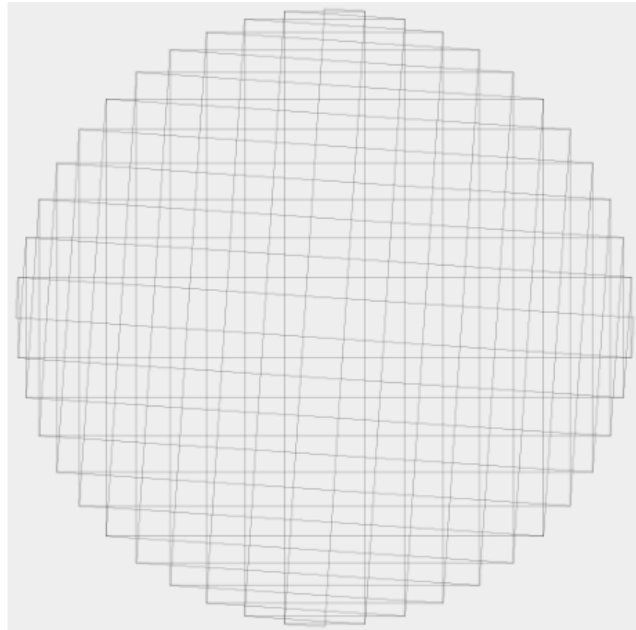
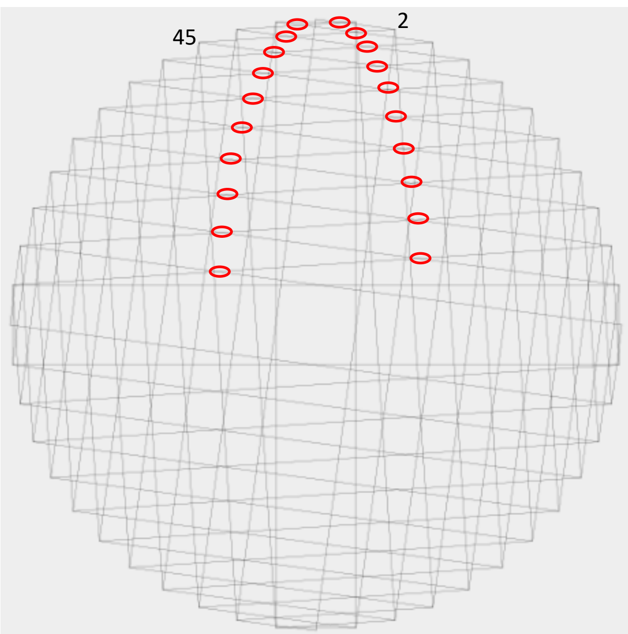
When do *Zig-Zag Corkscrews* and *Weaves* Appear and How do they Change?

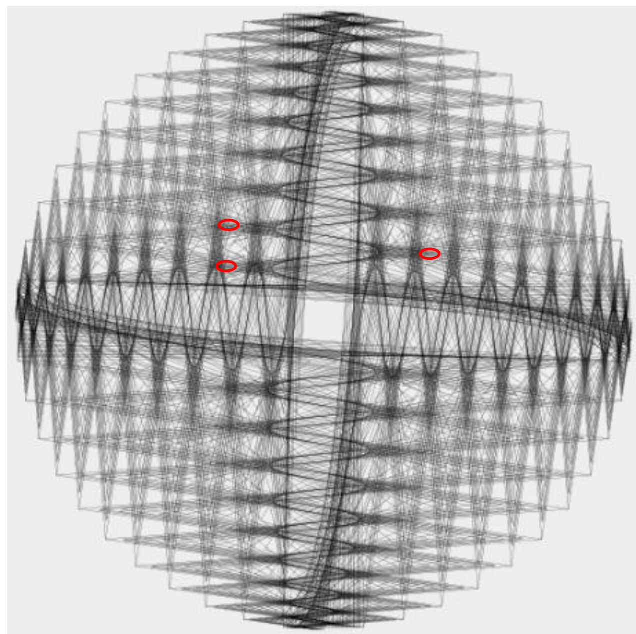
The image to the right is the 92-line VF for The Ultimate *Zig-Zag* Image sequence. The ***P*** = 113 = 5***S***-2 image from that section showed four corkscrews in the center of an oval cross. Here we explore why that occurred and where else it might occur.

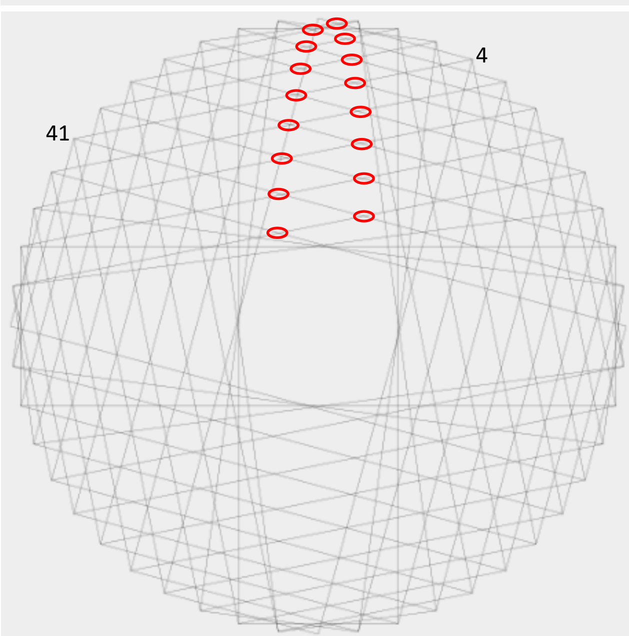


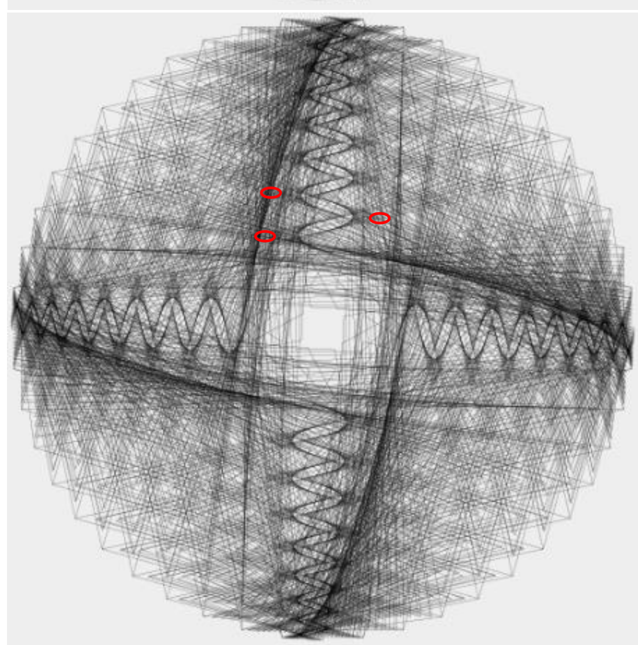
Consider only the 23 lines in the first jump set of the VF. There are 11 horizontal lines (smallest 1 to 47, largest 11 to 37) and 12 with *slope of 1* (meaning parallel to the lines connecting vertices 0-1 (smallest) to 37-12 (largest)). *Notice that there is NO interior intersection of these lines as they are simply the zig-zag VF.*

The middle row show ***P*** = 69 = 3***S*** and ***P*** = 65 and the bottom row shows ***P*** = 161 = 7***S*** and ***P*** = 159. The lefts have 92-lines, just like the VF (since ***P*** is a multiple of ***S***); the rights have 2116-lines.









Notice where the first two lines are drawn when *P* is a multiple of *S*. The counting rule requires that the first line of the image is ***P*** subdivisions along the VF. When ***P*** = ***S***, the image is the VF. But when ***P*** is a multiple ***m*** of ***S***, ***P*** = ***m***·***S***, the image uses only the vertices of the VF, but they are used in a different order. Given the *zig-zag jump set rule*, if ***m*** is even, ***m*** = 2***k***, and the first line will end up at vertex ***n***-***k***. If ***m*** is odd, ***m*** = 2***k***-1, and the first line will end up at vertex ***k***.

*Focus on odd **m** to create corkscrews*. In this instance, the second line ends at ***n***-***m*** since 2***m*** is even so long as the second jump remains in the first set. These two vertices are noted on the middle left and bottom left images.

Middle image: The first line is from 0 to 2 since ***P*** = 69 = 3***S*** and 3 = 2***k***-1 if ***k*** = 2. The second line is from 2 to 45 = ***n***-***m***.

Bottom image: The first line is from 0 to 4 since ***P*** = 161 = 7***S*** and 7 = 2***k***-1 if ***k*** = 4. The second line is from 4 to 41 = ***n***-***m***.

Internal intersections when *P* is an odd multiple of *S*. There are sets of lines parallel to the first two lines. Other lines intersect with these lines as well (due to 90° rotational symmetry) but focus on these two shallowly sloped directions. If ***m*** = 1, there are no internal intersections, but when ***m*** > 1, there are ***m***-1 internal intersections. The middle left image, ***m*** = 3, shows two internal intersections on each line and they are noted by the **red ovals**. The bottom left image, ***m*** = 7, has six internal intersections but the middle two on each line are noted by **red ovals**. If you count ovals along upward sloping lines, there are two each (or 20 total) on lines ending at vertices 1 through 10 in the middle panel, and 16 total on lines ending at vertices 1 through 8 in the lower panel. (If you create a similar graphic for ***P*** = 115 = 5***S*** you would need 18 such ovals.) *These central intersections form the outer corners of the curves that create the corkscrew.*

If an angle formed by two lines from two adjacent vertices intersect at another vertex (like the VF angles), then the angle is 180/***n*** = 180/48 = 3.75° according to the inscribed angle theorem. If the number of vertices between endpoints is larger than one, say ***m***, then the angle included is ***m***·180/***n*** = ***m***·180/48 = ***m***·3.75°. The same is true for interior angles created by connecting 4 vertices according to the interior angle theorem. This means that the angles in the middle and bottom left images at the red oval intersections are 11.25° and 26.25°, respectively.

What happens when *P* is close to odd multiples of *S*? If you scroll back and forth close to ***P*** = 69, 115, or 161 you will see that the images appear to have internal curves near the **red oval** points. The bottom three intersections in the middle and bottom right images have superimposed **red ovals** to orient you here. The curves near these points are not as precise as curved tip stars because the individual lines creating these curves are NOT part of the intersecting lines implied at the odd multiple of ***S*** values noted above. To be clear, note that the middle and bottom left images are NOT the VF despite having the same number of lines as the VF top image. As ***P*** increases near these multiples of ***S***, these "red oval curves" get larger and you can see a morphing of images that are like these multiple ***S*** images, even as ***P*** has SCF > 1.

What happens as *m* changes? Four things change as ***m*** = 2***k***-1 increases. **1.** The number of internal intersections increases (there are ***m***-1 intersections), therefore the distance between the middle two decreases and the corkscrew gets narrower. **2.** The number of sets of ovals decreases (there are 12-***k*** sets), therefore there are fewer corkscrew turns. **3.** The corkscrew angle gets greater (since the angle is ***m***·180/48°). **4.** The corkscrew gets harder to see (try ***P*** = 435).

Weaves*: *P* near 46*m*, an even multiple of *S*.** The final images show *weave patterns* based on multiples of 2S***. Two versions are possible: even, and odd ***m***. If ***m*** is even such ***P*** = 4·46 = 184 at far left, SCF = 92, the image has ***m***/2 diagonals, 23 lines, and vertex 23 at bottom. If ***m*** is odd such ***P*** = 7·46 = 322 at center right, SCF = 46, the image has ***m*** diagonals, 46 lines, and vertex 24 at bottom. In both cases, nearby ***P*** (189, center left and 325, far right) have ***m*** crossings in the weave.