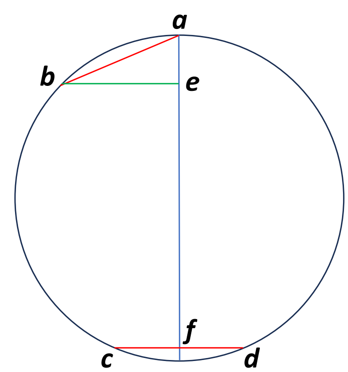
Why are Needles Created given S = 3, P = 4 for odd n and J = (n-1)/2?

The second ESA video provides visual "proof" that one obtains sharper and sharper needles as n increases given odd n values. Given (n,S,P,J) = (n,3,4,(n-1)/2), the first line drawn is from the top to a point one third of the way along the second line of the vertex frame from vertex (n-1)/2 to vertex n-1. This first endpoint is just to the right of the vertical diameter of the circle containing vertices of the n-gon and it forms the right side of the vertical needle in the final image (the left side of the vertical needle is the last line of the final image).

Since the n-gon is regular, each side has the same length. We focus our attention on two of those sides, the one just to the left of the top, from vertex n-1 to 0 and the one at the bottom from (n-1)/2 to (n+1)/2. For odd n, the bottom side is horizontal, and for all n, the side ending at the top is slanted.



The diagram to the right shows these two sides in **red** for an indeterminant odd n. In this context, b is vertex n-1, d is vertex (n-1)/2 and c is vertex (n+1)/2. Also shown in the vertical diameter in **blue**. Since the polygon is regular, ab = cd. Because n is odd, cd is horizontal. The inscribed angle theorem implies that angle abe = 180/n°. Much of our focus is on be shown here in **green**.

Given the above construction, first two lines of the vertex frame are ad and db and the first point of the final image is one third of the way from d to b. It remains to show that this point is to the right of the vertical diameter. The x-coordinate of this point is therefore one third of the way from d to b. Because abe is a right triangle, ab > be.

Since df = cd/2, the sum of horizontal lengths be + df is less than ab + df = 3df. If we note point b =(x_b,y_b) and d =(x_d,y_d), then the sum of horizontal lengths is $x_d+|x_b|$ (absolute value is necessary because x_b is a negative number).

The first drawn endpoint has x coordinate $x_d - (x_d+|x_b|)/3$. This endpoint is positive because $|x_b| < 2x_d$.

As n increases, the length of the leg ae decreases and the length of the leg be increases. As it increases, it approaches the length of the hypotenuse ab so that the first endpoint approaches the vertical diameter. Put another way, the needles get sharper and sharper as n increases.