## Kicking the Tires of an Image Sequence

One of the techniques discussed in ESA to understand the intricacies of image creation more deeply was to kick the tires of an image you like. This was discussed with regard to Three Shape-Shifting Triangles in E 8.6 and double jump models in E16.6. The idea posed there was to see what is essential in an image, and then vary other parameters of the model. The parameter that was varied in E8.6 was $\boldsymbol{J}$ (for fixed $\boldsymbol{n}, \boldsymbol{S}$, and $\boldsymbol{P}$ ), and in E16.6 it was $\boldsymbol{P}$ (for fixed $\boldsymbol{n}, \boldsymbol{S}$, and $\boldsymbol{J}$ ).

Here we focus on doing the same kind of analysis on image sequences. Since image sequences vary $\boldsymbol{P}$ for fixed $\boldsymbol{n}, \boldsymbol{S}$, and $\boldsymbol{J}$, $\boldsymbol{P}$ is no longer a parameter to be "kicked." Most of the interesting image sequences are made so by the structure of the jump set. The smoothness of morphing in an image sequence is determined by the number of images in the sequence and the size of $\boldsymbol{S}$. As discussed elsewhere, larger $\boldsymbol{S}$ and fewer primes in a full density image produce smoother sequences.

Another aspect of image sequences is that the most interesting sequences often are based on VCF $>1$. This allows a more open structure to the final image, even if that image is intricate and based on a reasonably large $\boldsymbol{n}$. Take for example, some images in the relative size of polygons discussion had $\boldsymbol{n}=32$ but VCF $=8$, and the ultimate zig-zag image discussion had $\boldsymbol{n}=48$ but VCF $=12$. Both situations lead to $90^{\circ}$ rotational symmetry and a notion of "square-ness." Similarly, GA2024 Section 5 examines equilateral three-ness using $\boldsymbol{n}=27$ and jump sets where VCF $=9$.

Given image sequences, $\boldsymbol{S}$ does not seminally affect the underlying images in the way that $\boldsymbol{n}$ and $\boldsymbol{J}$ do. Indeed, by adjusting $\boldsymbol{S}$ to align with the primes already involved in $\boldsymbol{n} / \mathrm{VCF}$ and $\boldsymbol{k}$ (the number of jumps in the jump sequence), one can reduce the fractional loss in number of images in the sequence due to images that are not full density. As a result, it is worthwhile to adjust $\boldsymbol{S}$ once $\boldsymbol{n}$ and $\boldsymbol{J}$ are set. Before examining this, we examine what happens as $\boldsymbol{n}$ adjusts given an interesting image sequence. We do this in the context of the 23 jump set ultimate zig-zag image sequence that was based on $\boldsymbol{n}=48$. It is worth remembering that if $\boldsymbol{J}>\boldsymbol{n}$, the actual jump is $\operatorname{MOD}(\boldsymbol{J}, \boldsymbol{n})$, so $\boldsymbol{J}_{2}=46$ means $\boldsymbol{J}_{\mathbf{2}}=1$ if $\boldsymbol{n}=45$.

Adjusting $\boldsymbol{n}$ so that VCF $=\boldsymbol{n}$. The sum of jumps in the set was $\mathrm{J}=540$ which has VCF $=\operatorname{GCD}(\boldsymbol{n}, \mathrm{J})=12$ as noted above. J has factors 54 and 10,45 and 12,36 and 15 , and 27 and 20 , among others. If any of these values is used for $\boldsymbol{n}$, then VCF $=\boldsymbol{n}$ and the images have no rotational symmetry. The number of lines in full density images in this instance is $529=23^{2}$, there are $506=529 \cdot(23-1) / 23$ such full density images, and each has porcupine $\boldsymbol{P}=264$. Two are shown below. The first two are $\boldsymbol{n}=45$ and the second two are $\boldsymbol{n}=36$ with the left of each pair being $\boldsymbol{P}=13$ and right $\boldsymbol{P}=264$.


VCF < $\boldsymbol{n}$. At left, $\boldsymbol{n}=24, \mathrm{VCF}=12,1058$-line $\boldsymbol{P}=45$ and 65 which is a single corkscrew. At right, $\boldsymbol{n}=50, \mathrm{VCF}=10,2645$-line $\boldsymbol{P}=13$ and 666. The VF of the right images are swirling needles created because 50 is slightly larger than zig-zag $\boldsymbol{n}=48$.




Modifying the Jump Set. An additional jump could have been added to the ultimate zig-zag image. If the $24^{\text {th }}$ jump of 24 is added, one final pair of lines are added to the VF, horizontal and vertical diagonals shown to the left. The resulting image sequence is not as smooth because there are only 704 2208-line full density images in the sequence and even if you increase $S$ to 24 there are only 7682304 -line full density images, or 16 images in each of 48 waves, 8 going in, 8 going out rather than 11 in and 11 out with 23 jumps and $\boldsymbol{S}=23$. Nonetheless, adding this final jump provides interesting image sequence possibilities as $\boldsymbol{n}$ changes.

Reducing $\boldsymbol{n}$ by 1 to 47 , for example, creates the stunning 24 -line VF at right because each pair of jumps in the zig-zag model sum to 47 . The result is a spray of 12 lines, one each from 0 to odd vertices $1,3,5, \ldots, 23$ and back. The resulting image sequence has 192576 -line full density images.

The next two examples both have VCF $=\boldsymbol{n} / 3$ or $120^{\circ}$ rotational symmetry and 5761728 -line images per sequence. The two at left are based on $\underline{n=36}$. The first shows curved tip stars with very small curves, $\boldsymbol{P}=7$, to not obscure the fact that three vertices have no curves at their tips. They form an inverted equilateral triangle (i.e., flat top) of vertex 6,18 , and 30 (although these vertices are not connected in the image). The reason that they do not have curves is that the $12^{\text {th }}$ jump is a jump of 36 , the same as a zero jump as discussed in E17.4.2. The angle $(\angle)$ at each of these vertices is $60^{\circ}$ since the $11^{\text {th }}$ jump is 11 and the $13^{\text {th }}$ jump is 13 meaning the angle created by these vertices is $12=\boldsymbol{n}-11-13$ or one third of the circle. Extending the lines from each of the 11 sides completes these three truncated equilateral triangles. Note that some of the angles are $55^{\circ}$ (for example, $\angle 0,12,25$ ), and $65^{\circ}$ (for example, $\angle 12,25,35$ ), but most are $60^{\circ}$ including the lone inscribed equilateral triangle in the image connecting vertices 0 to horizontal base 12 and 24 . Note also that the VF shows three sharp triangles (connecting $(0,12,13),(12,24,25)$ and $(24,0,1)$ each of which are $\left(5^{\circ}, 115^{\circ}, 60^{\circ}\right)$ triangles). The second image, $\boldsymbol{P}=481$ shows the clear triangular structure of the image with a burst of lines emanating from the zero jump vertices $(6,18,30)$ noted above. The two images at right are based on $n=18$. The images chosen are close factors of porcupines. The first is $\boldsymbol{P}=$ porcupine $/ 3=287$ (288 is a triangle) and second is $\boldsymbol{P}=$ porcupine $/ 2=431$ ( 432 is a 4-line twice drawn vertical line). Due to the 6,2 -star, you might think the first has $60^{\circ}$ rotational symmetry, so look closely.


The last four images are from $n=16$ with VCF $=4$. The first, $\boldsymbol{P}=65$, shows crossing corkscrews. Subsequent waves show decreasing amplitude corkscrews just as discussed elsewhere. The second is $\boldsymbol{P}=575=$ porcupine $/ 2$ ( 576 is a square). The last two, $\boldsymbol{P}=745$ and $\boldsymbol{P}=841$ differ by $4 \boldsymbol{S}$ and show the woven structure of the crossed zig-zags in this situation.


