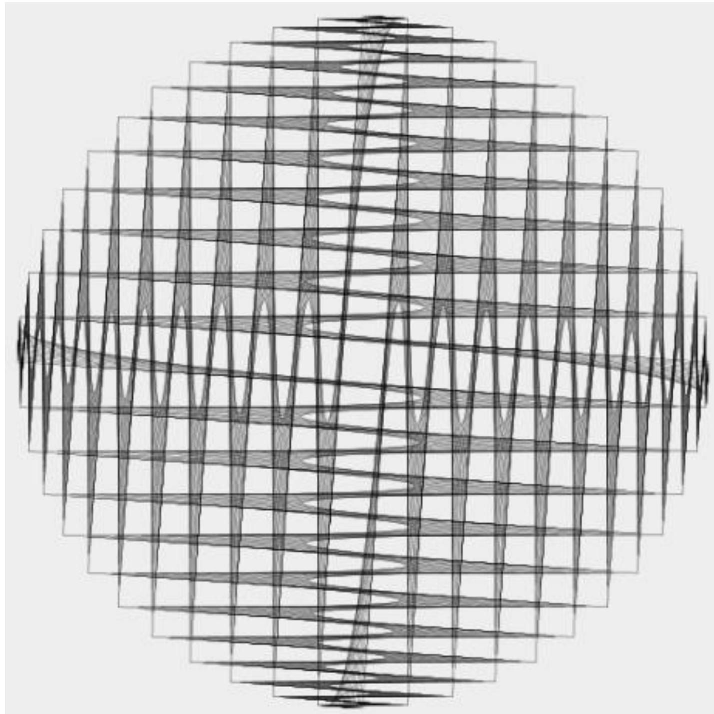
The Ultimate *Zig-Zag* Image



The image to the right shows ***P*** = 19, for ***n*** = 48 and ***S*** = 23. The key here is the jump set pattern which can be seen given this curved-tip star (***P*** < ***S***). There are 23 jumps in all, with jump ***i***, J_i, created according to the *Zig-Zag Rule*:

Zig-Zag Rule: If ***i*** is odd, $J_i = i$, if ***i*** is even, $J_i = n - i$.

(Reverse odd and even and the zig-zag becomes positively sloped.) The resulting vertex frame, VF, jumps back and forth in zig-zag fashion, much as was done elsewhere:

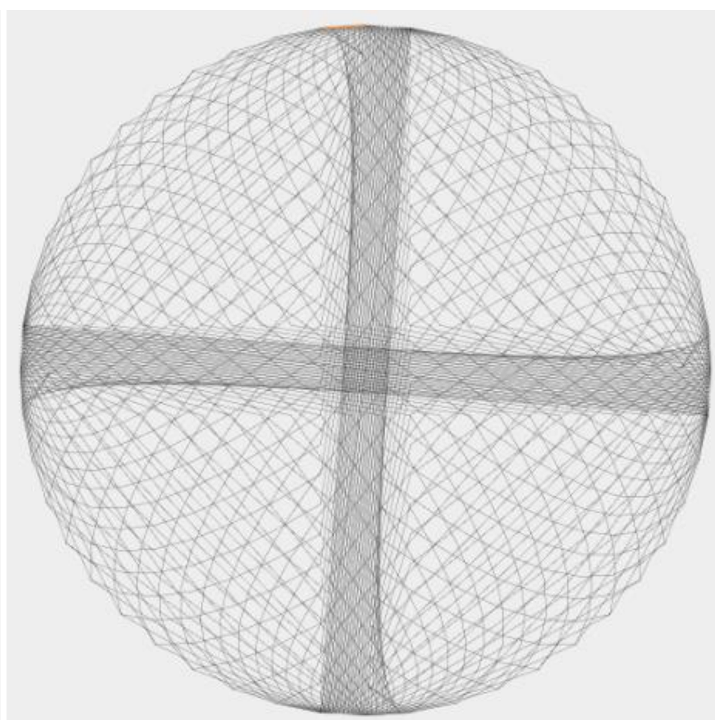
Jump pattern: 1, 46, 3, 44, … , 21, 26, 23

48-gon vertex: 0, 1, 47, 2, 46, … , 11, 37, 12

The sum of jumps in the jump set is 540 which means that VCF = 12 and the jump set ends one quarter of the way around the circle at vertex 12 = MOD(540,48). This means that there is 90° rotational symmetry and a total number of lines **N** = $4 \cdot 23^2$ = 2116 per full density image.

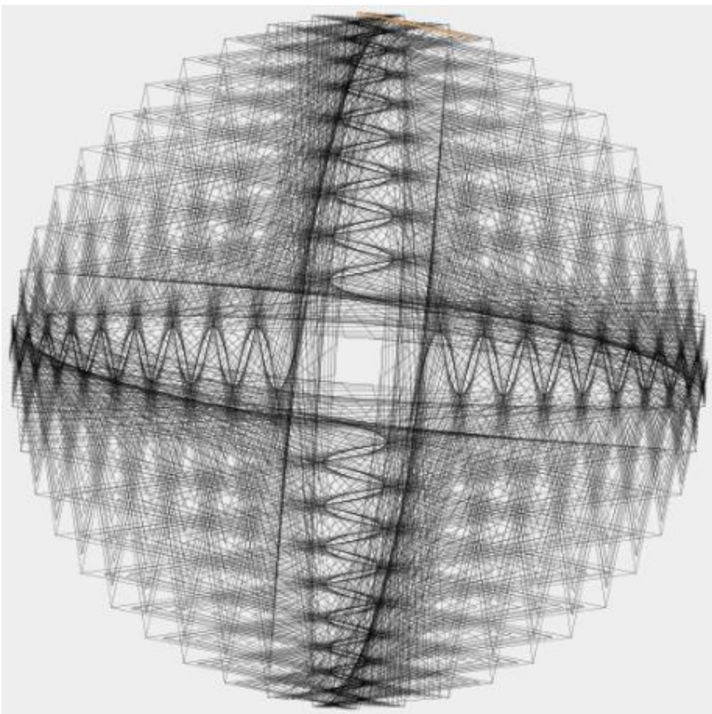
Given prime factors 2 and 23 of **N**, the image sequence has **T** = 1012 = 2116·(2-1)/2·(23-1)/23 full density images using the rule discussed here. Half of these images are distinct because they are symmetric about the *porcupine* ***P* = 1057**.

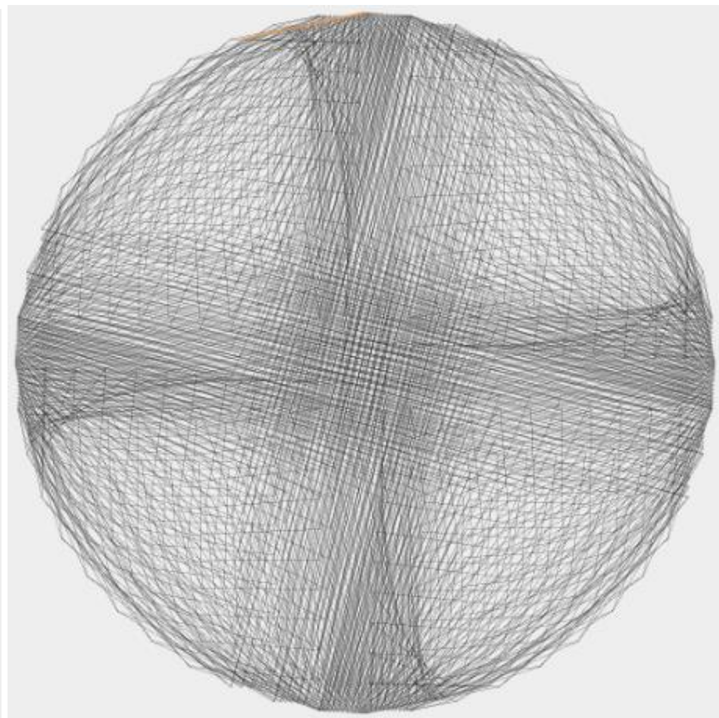
Given this number of images, the sequence takes about 2 minutes to watch at the default speed. It provides a very nice sequence of waves but one of the things that happens very quickly is that the image appears to be a cross with webbed areas outside the cross at the end of the first wave. A bit of exploration produces this stunning observation. When ***P*** = 46, one obtains the 46-line image on the left (since SCF = 46); only vertices of the 48-gon are used (***P*** = 2***S***). (For reference, the first line of the image from vertex 0 to 47 is shown in orange here using the *Home* mode.) When ***P*** = 45, the 2116-line image on the right appears (and ***P*** = 47 is very similar). Similar but not quite the same things happen when ***P*** is near additional multiples of 46. As ***P*** increases across these multiples, the size of the internal square increases in size.



Why the waves wash in and out. If you simply let the overall sequence play out a few times you will notice the beating nature of the image sequence. If you take the time to count, there are 46 waves with 22 images per wave, 1012 = 22·46. Since ***S*** = 23, there are 11 odd values (meaning different full density images) on each line of the VF. Due to the zig-zag nature of the VF, values near even multiples of 23 look a lot more like one another, than values near odd multiples of 23.

Put another way, ***P*** = 113 = 5·23-2 at left is in the middle of the third wave but ***P*** = 137 = 6·23-1 is at the end of that wave. Notice the similarity between ***P*** = 19 above and the one on the left, versus ***P*** = 45 and ***P*** = 113. There are three central squares before the zig-zagging starts with P = 113, but only 1 with P = 19, nonetheless, the crossed nature of the zig-zags seems quite similar. The same goes for the start and end of other multiples of 46. The first half of each wave moves toward greater zig-zag visibility, the second half toward less.





The two final images to consider are the near *porcupine*/4 = ***P*** = 263 at left, and the near *porcupine*/2 = ***P*** = 527 at right.