MA. A Modular Approach to Swirls

If ***n*** and ***J*** < ***n***/2 are coprime, then there will be a number ***a*** that is the modular multiplicative inverse, MMI, of ***J*** mod ***n***. We say that ***a*** is the MMI of ***J*** mod ***n***. This means there exists a multiple ***b*** of ***n*** such that ***a*·*J*** = ***b*·*n***+1.

Cross-multiplying, we see the following: ***a*/*b*** = ***n*/*J***+1/(***b*·*J***) or ***a*/*b*** > ***n*/*J***

The numbers ***a*** and ***b*** can be obtained by Backtracking Euclid's Algorithm as discussed in E24.3 (using the *Excel* file discussed there).

One can also use this methodology to obtain the negative modular multiplicative inverse, nMMI, or value that is 1 less than a multiple of ***n*** as discussed in E24.2. All that needs to be done is to replace ***J*** with ***n*-*J*** in the backtracking algorithm.

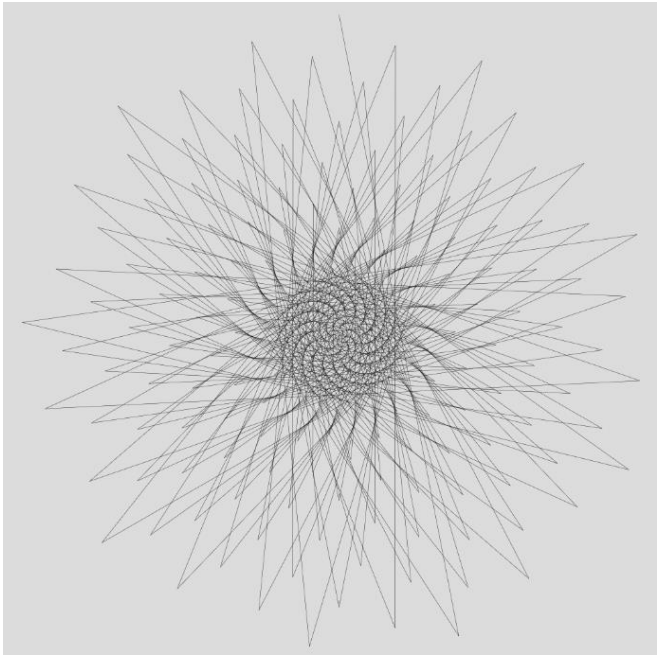
An example. ***n*** = 57, ***J*** = 20.

MMI. 20 is the MMI of 20 mod 57 because 20·20 = 400 = 7·57+1 = 399+1, or ***a*** = 20 and ***b*** = 7. In terms of swirls, the 20th line ends on the vertex 1 radius, or the 20,7-swirl is clockwise, ↻, just as shown in the upper right image of *Competing Swirls*.

nMMI. 37 is the nMMI of 20 mod 57 because 37·20 = 740 = 13·57-1 = 741-1, or ***a*** = 37 and ***b*** = 13. In terms of swirls, the 20th line ends on the vertex 56 (or -1) radius, or the 37,13-swirl is counter-clockwise, ↺, just as shown in the lower left image of *Competing Swirls*.

This offers an analytical, as opposed to a geometric, way to find the number of line segments required before the end of the last line is on the vertex 1 radius or the vertex ***n***-1 radius.

A second example. The ***n*** = 66, ***J*** = 25, ***r*** = 200 image at right involves four distinct swirls. The height of vertices near 0 along the vertex radii from -3 to 2 suggest that the values from smallest ***a*** to largest ***a*** will be -3, 2, -1, 1, -2. The above discussion will help us find ***a*** for -1 and 1.



The values in the table below were obtained visually for the vertex 63 (or -3) radius which quite clearly forms the ↺ inner swirl. The middle ↻ swirl is based on a twisted 16,7-star which ends at the vertex 2 radius. The outermost 41-18-swirl is ↻ and not as prominent as the ↺ 25-11-swirl.

Note that -2 is not listed in the table as it is simply the second iteration of the *Outer 1* swirl. Line 50 ends on the vertex -2 radius.

Parent		Outer 1	Outer 2	Inner	middle
66	***n*** ***a***	25	41	9	16
29	***J*** ***b***	11	18	4	7
2.2759		2.2727	2.2778	2.25	2.2857
	Curl	**CCW**	**CW**	**CCW**	**CW**
Vertex Radius		65, -1	1	63, -3	2