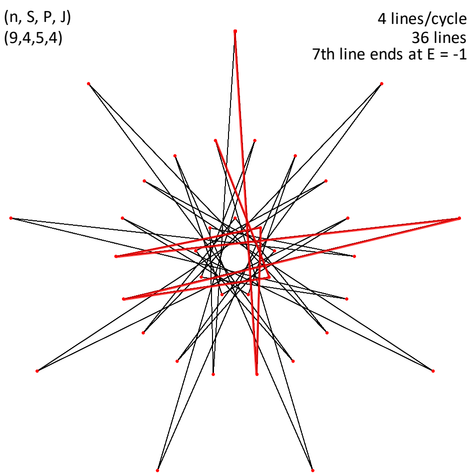
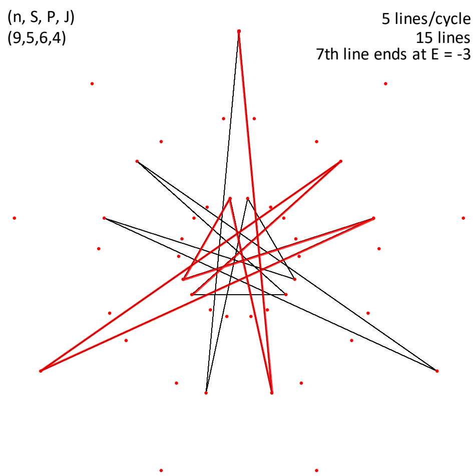
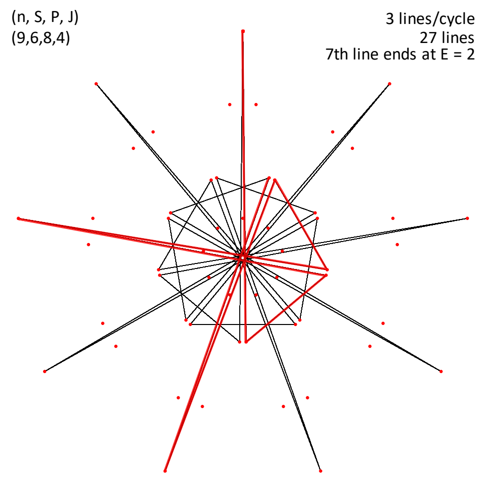
Multiples of *S* and the Limits of Image Types

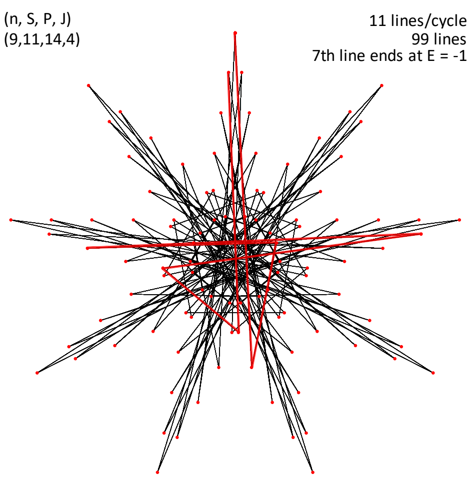
In E9.6 we argued that there are often lower limits to image types. The images below include subdivision dots and the first 7 lines overlaid in **red**. They provide an example of the lower limits issue as well as a springboard for discussing how images change as ***S*** changes.

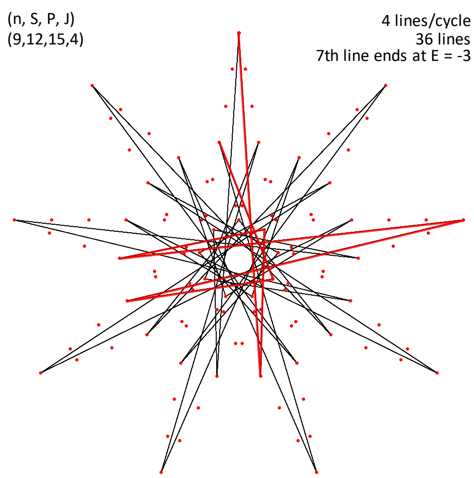
When using *The 7-Line Generator Function* we obtain an image in which the end of the 7th line is within 3 subdivisions of the top. If ***S*** is not divisible by 7, one can consider ***S*** as one of six values within ±3 of an ***S*** divisible by 7 (much like we examined 18 ≤ ***S*** ≤ 24 on either side of ***S*** = 21 here). These six ***S*** values come in three pairs, two each of images where the 7th endpoint, ***E***, is within 1, 2, and 3 of the top. (Not all have SCF = 1 as we see looking at the six images below which are 3, 2, and 1 below ***S*** = 7 (top) and 14 (bottom) for ***n*** = 9 and ***J*** = 4). Those that are ***E*** = ±1 are single-step of length 7 images (like the ***E*** = -1 left column) and those that are ±2 or ±3 may have SCF > 1 but may be smallest-step even though they are not single-step (top middle is not smallest-step, but bottom middle is smallest-step, both ***E*** = -3). The right column with ***E*** = 2, is smallest-step at top since SCF = 2 but is second smallest-step at bottom since SCF = 1 but the 7th line ends at subdivision endpoint 2. Each is a variation on the left column discussed here.

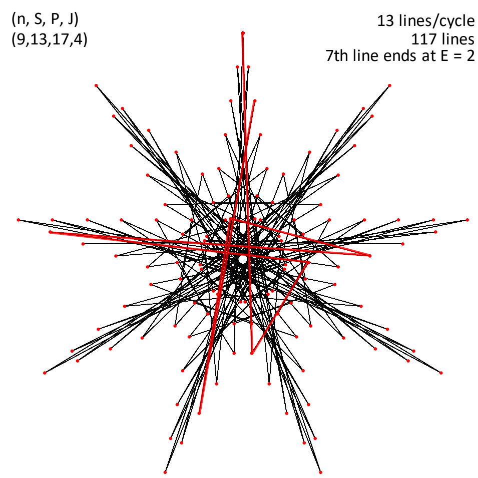




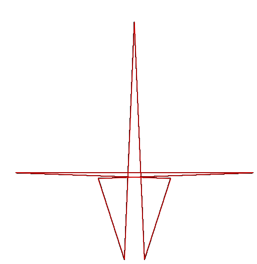








The limits on image types. The middle and right in the top row are not obvious versions of the sharply pointed stars that occur based on the 7-line zero-remainder *glider* image shown to the right. The distorted wings of the glider when ***S*** is not divisible by 7 create images that resemble a series of ***n*** Christmas trees arrayed around the center when ***S*** is large (like the ***n*** = 9 trees when ***S*** = 38 shown here), but when ***S*** is too small, the distortions become too great, and different kinds of images emerge.



There are two possibilities when |*E*| > 1. If |***E***| > 1, either SCF = 1 or SCF = |***E***|. SCF cannot be larger than |***E***| because that would mean that the first subdivision endpoint used on the VF after the top (and the last one used on the last line of the VF) would be more than 3 subdivisions away from the top (which cannot happen because we know that |***E***| is within

3 of the top). Both other possibilities can occur: the top right image shows SCF = |***E***| = 2; the bottom right shows SCF = 1. The SCF = |***E***| version deserves additional attention.

What happens when SCF = |*E*| > 1? Two possibilities arise, both of which are represented by the middle and right images in the top row. Assume you have fixed ***n*** and ***J*** < ***n***/2 so that the VF is fixed and additionally assume that VCF = 1 so that the VF has ***n*** vertices. The two possibilities are that ***S*** is not divisible by SCF or ***S*** is divisible by SCF.

S** is not divisible by SCF*. The image will not reappear as ***S changes if ***S*** is not divisible by SCF. Consider the top middle image with ***S*** = 5 and ***P*** = 6, ***E*** = -3 and SCF = 3 because of the commonality between ***P*** and ***n*** = 9. One cannot reduce ***P*** and ***S*** by a factor of 3 because ***S*** is not a multiple of 3. And multiplying ***S*** and ***P*** by 2 to ***S*** = 5, ***P*** = 12 produces ***E*** = -6 but this is not within 3 of the top and the ***P*** generation function produces ***P*** = 13 with ***E*** = 1.

S** is divisible by SCF.* If ***S is divisible by SCF the image will appear a total of three times as ***S*** changes. If we call the initial version S_0, let $S_1 = S_0$/SCF. This will necessarily have |***E***| = 1 and it will be the same image with SCF = 1. Let $S_2 = 2 \cdot S_1$, and $S_3 = 3 \cdot S_1$ (note that one of these will be S_0). *The 7-Line Generator Function* ***P*** values in this instance will be 2 and 3 times the ***P*** value for S_1, and |***E***| will be 2 and 3, and the same image results. The only difference will be SCF = 2 and 3 as well.

Two interrelated examples. In the context of the upper right image, S_0 = 6 and SCF = 2 so S_1 = 3 (so ***P*** = 4), S_2 = 6 (so ***P*** = 8), and S_9 = 9 (so ***P*** = 12) produce the same image. The only difference is ***E*** = 1 and SCF = 1 with S_1, ***E*** = 2 and SCF = 2 with S_2, and ***E*** = 3 and SCF = 3 with S_3. If we check the next multiple of 3 for ***S***, ***S*** = 12, we obtain a different image because ***E*** > 3 is not allowed given this ***P*** generator function. With ***S*** = 12, we obtain the middle bottom image which has ***P*** = 15, ***E*** = -3 and SCF = 3 (not ***P*** = 16 and ***E*** = 4). In fact, this image is THE SAME as the top left image, ***S*** = 12/SCF = 4 (so ***P*** = 5), has ***E*** = -1 and SCF = 1. The only difference with the bottom middle image is that only one third of the subdivision **dots** are used to create the image since SCF = 3. Note that this is the third ***S*** that produces the same image. The second ***S*** that produces this image, not shown but easily understood, is when ***S*** is double the top left SCF = 1 value, ***S*** = 8 (so ***P*** = 10), ***E*** = -2 and SCF = 2.

Multiples of *S*. *If an image is single-step and if **P** is generated by The 7-Line Generator Function, then there are three values of **S** that produce the same image but only the smallest **S**, the |**E**| = 1 version, is single-step.*

This is in marked contrast to traditional string art in which multiplying ***S*** and ***P*** by a constant ***c*** produces the same image for every whole number ***c***. The only difference is that SCF will be ***c*** times as large as the original SCF for the image. The difference here is that ***P*** is functionally related to ***S*** via *The 7-Line Generator Function* and hence will change after three iterations (for ***c*** = 1, 2, 3 if the original (***n***,***S***,***P***,***J***) had |***E***| = 1).