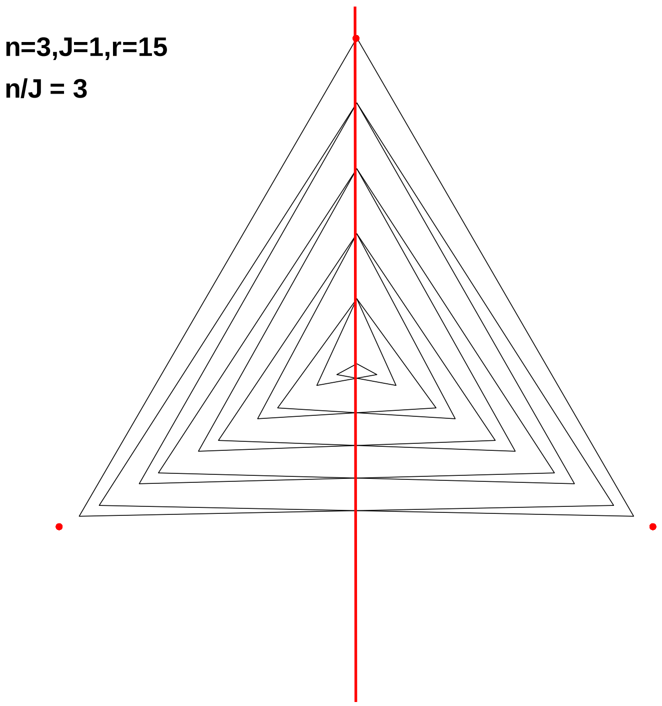
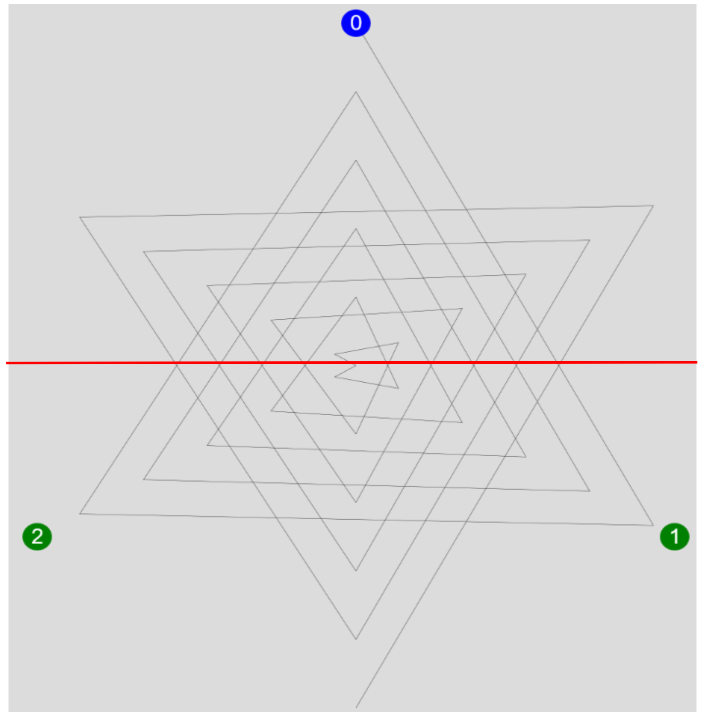
On the Web Mirror Line of Symmetry and When Will it Coincide with the *Excel* Mirror?

We saw in *Two Types of Spiral Mirrors* that the *Excel* mirror is always symmetric about the vertical diameter. The example shown there used the simplest possible polygon (a triangle) to show how the web mirror works. That example had a horizontal diameter mirror. Quite clearly, these mirrors do not line up with one another, the *Excel* mirror, with vertical symmetry is to the left, and the image to the right reproduced from the previous section. The *Line of Symmetry*, LoS, for both images is shown in **red**.





Location of the Web Spiral Final Endpoint. Define the *final used vertex* as **F**. This point occurs after 2·***r*** lines with ***J*** vertex jumps per line around the parent ***n***-gon. Therefore, **F** is located at **F** = MOD(2·***r***·***J***, ***n***). The location of the final endpoint is a unit away from the center at a point opposite **F**, or at a point on the unit circle at **F**+***n***/2. If ***n*** is even, this is one of the vertices of the ***n***-gon, but if ***n*** is odd, it is halfway between two vertices, like at right given **F** = MOD(30,3) = 0 and ***n*** = 3.

Location of the Web Spiral LoS. As ***n***, ***J***, and ***r*** vary, the LoS in the web version varies. That line must pass through the origin so one of the two points needed to describe the line is already determined. The other point is midway between the start of the spiral (at vertex 0) and the end of the 2r^{th} line (at **F**+***n***/2), or at point (**F**+***n***/2)/2 = **F**/2+***n***/4. Of course, this need not be a vertex of the parent ***n***-gon. At right above the LoS is at 1.5/2 = 3/4, or at 3-o'clock on a clockface.

When will the Web LOS coincide with the *Excel* LoS? The web LoS will be vertical if **F**/2+***n***/4 = ***n***/2. Simplifying, this means that if **F** = ***n***/2, the LoS will be vertical. Put another way, if ***n***/2 = MOD(2·***r***·***J***, ***n***), then the LoS will be vertical.

We can unpack this a bit more. We know that ***n*** must be even to have the final endpoint to occur at a vertex of the ***n***-gon. If ***n*** is divisible by 2 but not 4, then the bottom of the ***n***-gon, ***n***/2, is an odd number. To have the final endpoint at 0, **F** must be at ***n***/2. But **F** is a jump of 2·***r***·***J*** vertices which is necessarily even, regardless of how many times around the ***n***-gon is this number of vertex jumps. If we subtract off those multiples of ***n*** (with ***n*** even), we are left with the fact that **F** must an even number of vertices. To have a vertical mirror, ***n*** must therefore be divisible by 4.

The requirement that ***n*** is divisible by 4 is necessary but not sufficient for the web version to have a vertical mirror. The sufficient condition, as noted above, is that ***n***/2 = MOD(2·***r***·***J***, ***n***). Note that in this setting, if ***J*** is even, then the spiral no longer uses all its vertices. This table provides a summary of what you will find for ***n*** = 16 for ***J*** = 1 to 6 in terms of final endpoint and line of symmetry (vertical LoS are bolded in the table since 8 = ***n***/2 and 4 is a horizontal LoS here).

16 =***n*** 1 =***J*** ***r***=	**4**	5	6	7	8	9	10	11	**12**	3 =***J*** ***r***=	**4**	5	6	7	8	9	10	11	**12**	5 =***J*** ***r***=	**4**	5	6	7	8	9	10	11	**12**
Final vertex, **F**	**8**	10	12	14	0	2	4	6	**8**		**8**	14	4	10	0	6	12	2	**8**		**8**	2	12	6	0	10	4	14	**8**
Ending point, **F**+***n*** /2	**16**	18	20	22	8	10	12	14	**16**		**16**	22	12	18	8	14	20	10	**16**		**16**	10	20	14	8	18	12	22	**16**
Symmetry contains	**8**	9	10	11	4	5	6	7	**8**		**8**	11	6	9	4	7	10	5	**8**		**8**	5	10	7	4	9	6	11	**8**
16 =***n*** 2 =***J*** ***r***=	4	5	**6**	7	8	9	**10**	11	12	4 =***J*** ***r***=	4	**5**	6	**7**	8	**9**	10	**11**	12	6 =***J*** ***r***=	4	5	**6**	7	8	9	**10**	11	12
Final vertex, **F**	0	4	**8**	12	0	4	**8**	12	0		0	**8**	0	**8**	0	**8**	0	**8**	0		0	12	**8**	4	0	12	**8**	4	0
Ending point, **F**+***n*** /2	8	12	**16**	20	8	12	**16**	20	8		8	**16**	8	**16**	8	**16**	8	**16**	8		8	20	**16**	12	8	20	**16**	12	8
Symmetry contains	4	6	**8**	10	4	6	**8**	10	4		4	**8**	4	**8**	4	**8**	4	**8**	4		4	10	**8**	6	4	10	**8**	6	4