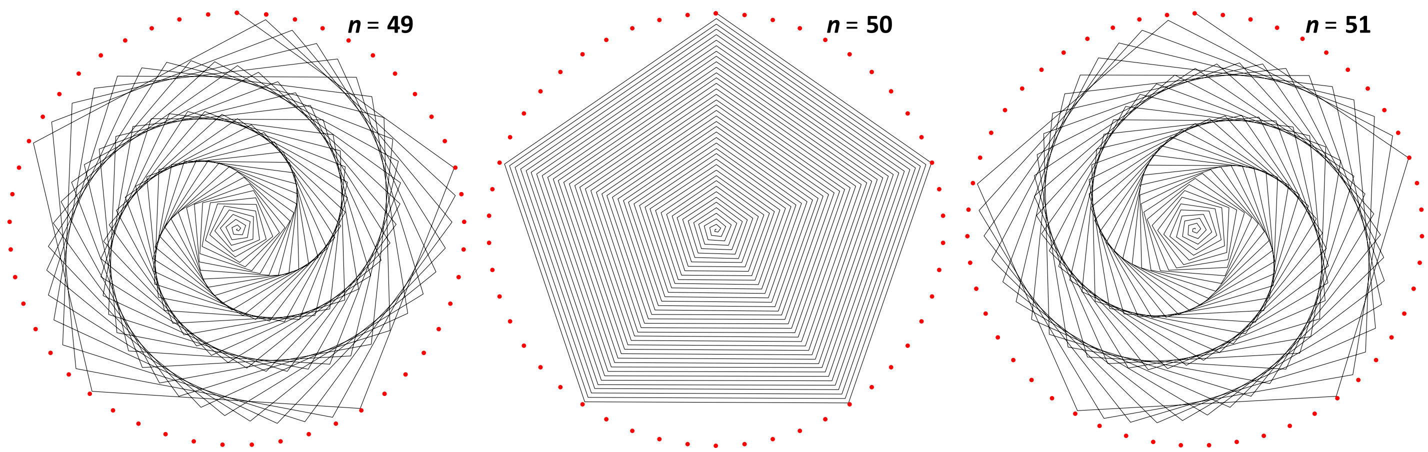
Simple Swirls from Almost Polygons

Almost polygons occur when n is a multiple of J. For example, the middle image is an almost pentagram based on $J = 10$, the multiple $a = 5$, so $n = a \cdot J = 50$. The fifth line ends after five jumps of 10, or at vertex 0 if $n = 50$. It is not an exact pentagram because the fifth line ends $5/r = 1/40$ in from the start of the first line on the vertex 0 radius given $r = 200$.



Spirals versus Swirls. In this chapter, we distinguish between spirals and swirls. A spiral involves how individual lines rotate around the parent polygon. If $J < n/2$, the image is a clockwise, ↻, spiral, and if $J > n/2$ the image is a counter-clockwise, ↺, spiral. By contrast, *a swirl involves the overall rotational aspect of the image as your eyes move towards the center*. All three images above are ↻ spirals because $10 = J < n/2$ but they differ according to how each image swirls. The left is a clockwise swirl, the right is a counter-clockwise swirl, and the middle has no swirl.

Clockwise, , swirls. If n is one less than a multiple, a, of J, then the a^{th} line ends on the vertex 1 ray. The $2a^{th}$ line ends on the vertex 2 ray, and so on. The left image above is an example of this, for $a = 5$. Note that such *overly-closed* polygons produce swirls.

Counter-clockwise swirls, , swirls. If n is one more than a multiple, a, of J, then the a^{th} line ends on the vertex n-1 ray. The $2a^{th}$ line ends on the vertex n-2 ray, and so on. This is the situation shown on the right above, for $a = 5$. Note that such *cracked-open* polygons produce swirls.

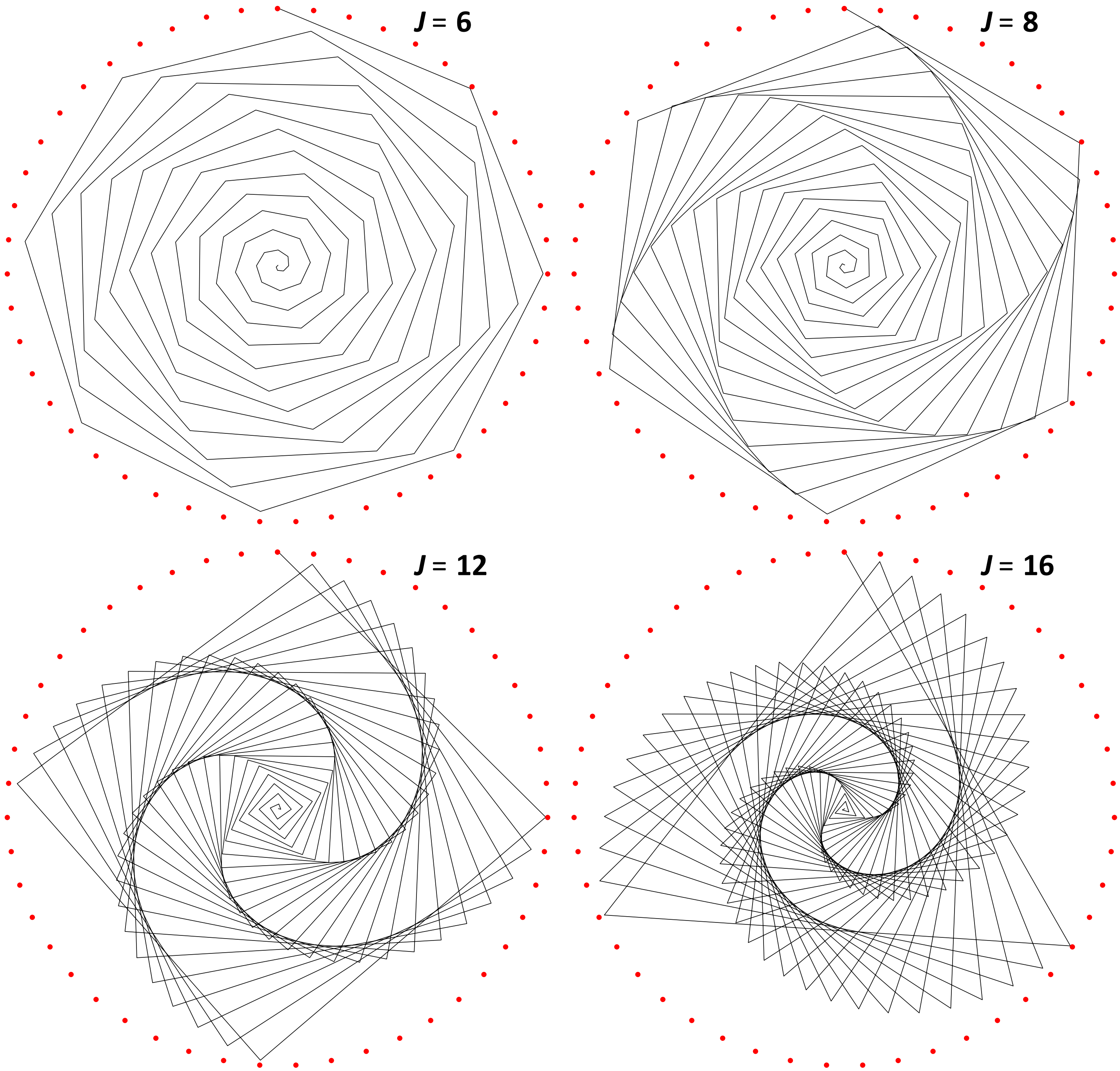
There are *a* spines in the swirl. You can think of the sides of the initial a-gon as providing the open ending of an individual curve in the swirl. One can think of these curves as spines of the swirl. In this instance, the first 5 lines begin to outline the image on the left. There are five spines in this image. 1) Focus on the lines that are of the form $5k+1$ for $k = 0, 1, 2, \ldots$ (i.e., the 1st line, 6th line, 11th line, and so on). These lines form the outline of the spine that starts at vertex 0 and rotates into the center. 2) The lines of the form $5k+2$ for $k = 0, 1, 2, \ldots$ form the outline of the spine that starts at vertex 10 and rotates into the center. 3) The lines of the form $5k+3$ for $k = 0, 1, 2, \ldots$ form the outline of the spine that starts at vertex 20 and rotates into the center. 4) The lines of the form $5k+4$ for $k = 0, 1, 2, \ldots$ form the outline of the spine that starts at vertex 30 and rotates into the center. 5) The lines of the form $5k+5$ for $k = 0, 1, 2, \ldots$ form the outline of the spine that starts at vertex 40 and rotates into the center. 1)-5) show that there are $a = 5$ curved spines that cumulatively create the swirling image at left above.

We could have produced a similar analysis for the five spines in the right image above, but care would have to be exercised since now we are talking about swirling spines.

An alternative way to conceptualize these spines is to think of the image as a set of a spiral staircases that swirl to the center. (The term *spiral staircase* uses the word *spiral* here is common jargon, we would prefer to call this a swirling staircase.) The steps of the staircase are the "other" line attached to each endpoint, since only half of each line is used for the spine (or banister), the other is the step (although it is also part of the banister for another part of the image). To put this in the context of the left image above, the first half of the 5^{th} line is the start of the 5^{th} banister rail (starting at vertex 40), but the last half of the 5^{th} line is the “first” step in the 1^{st} staircase (which had banister starting at vertex 0).

As these spines approach the center, they eventually disappear because the twisted ***a***-gons no longer overlap. One can follow the curves by noticing where the lines kink, but without the nested-twisted-polygons overlapping or touching, the resulting curves become harder to distinguish.

For fixed *n*, we can see different numbers of spines in the swirl. The four images below have ***n*** = 47 and vary by ***J*** so that each produces swirls. Each has ***r*** = 100 so that the swirling steps remain visible for the lower right triangular image (set ***r*** ≥ 180 to obtain superior ***J*** = 6 upper left images because the initial twisted-octagons overlap with one another).



The number of spines is easiest to count if you focus on the outside of the image. ***J*** = 6 is an octagon so there are 8 spines, ***J*** = 8 is a hexagon with 6, ***J*** = 12 is a square with 4, and ***J*** = 16 is a triangle with 3. Each is *overly-closed* because in each case, ***a·J*** = 48, which is why the a^{th} line ends on the vertex 1 radius. If you change to ***n*** = 49, then each of these ***J*** values would produce images with swirls because then the a^{th} line ends on the vertex 48 radius.